**Math 432: Set Theory and Topology** HOMEWORK 1 Due date: Jan 26 (Thu)

**Exercises from Kaplansky's book.**

**Sec 1.1:** 8

**Sec 1.2:** 8

**Sec 1.4:** 7 (HINT: It is enough to prove that $f$ is invertible, i.e. define an inverse.)

**Other** (mandatory) **exercises.**

**1.** Deduce the identity $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ from the distributivity law in the following two ways: using complements and not using complements (directly applying distributivity).

**2.** Recall that $A \triangle B := (A - B) \cup (B - A)$ and prove the following (using any method you like).

(a) $A \triangle B = (A \cup B) - (A \cap B)$.

(b) $A \triangle C \subseteq (A \triangle B) \cup (B \triangle C)$.

**3. Terminology.** For a function $f : A \to B$ and $A_0 \subseteq A$, let $f\downharpoonright_{A_0}$ denote its *restriction to* $A_0$, namely, the function $f\downharpoonright_{A_0} : A_0 \to B$ defined by $f\downharpoonright_{A_0}(a) := f(a)$ for each $a \in A_0$. When we say "$f$ on $A_0$ has some property", we mean that $f\downharpoonright_{A_0}$ has that property.

Let $f : A \to B$ and $g : B \to C$.

(a) Prove that $g \circ f$ is injective if and only if $f$ is injective and $g$ is injective on $f(A)$ (i.e. $g\downharpoonright_{f(A)}$ is injective).

(b) Give an example of $f$ and $g$ such that $f$ is injective yet $g \circ f$ is not.

(c) Prove that $g \circ f$ is surjective if and only $g$ is surjective on $f(A)$ (i.e. $g(f(A)) = C$).

(d) Give an example of $f$ and $g$ such that $g$ is surjective yet $g \circ f$ is not.

**4.** Let $f : A \to B$ and $A_0, A_1 \subseteq A, B_0, B_1 \subseteq B$.

(a) Prove that $f^{-1}$ respects unions, i.e. $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$.

(b) Prove that $f^{-1}$ respects complements, i.e. $f^{-1}(B_0') = f^{-1}(B_0)'$.

(c) Prove that $f$ respects unions, i.e. $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$.

(d) Show that $f(A_0') \subseteq f(A_0)'$ and $f(A_0') \supseteq f(A_0)'$ don't hold in general by constructing a counterexample to each.