# MATHEMATICAL LOGIC

MATH 570 Anush Tserunyan FALL 2017

443 Altgeld Hall 2–3:20pm TueThu

The course will introduce the main ideas and basic results of mathematical logic from a fairly modern prospective, providing a number of applications to other fields of mathematics such as algebra, algebraic geometry, and combinatorics. It will consist of three parts: basic model theory, basic recursion theory, and more.

**Basic model theory.** Model theory is a study of mathematical structures, examples of which include groups, rings, fields, graphs, and partial orders. We will first abstractly study structures and definability, theories, models and categoricity, as well as formal proofs, and this will culminate in proofs of the Gödel Completeness and Compactness Theorems—two of the most useful tools of logic. Then, we'll apply the developed techniques to concrete examples such as the structure of natural numbers and algebraically closed fields; the latter will yield a rigorous proof of the Lefschetz Principle (a first-order sentence is true in the field of complex numbers if and only if it is true in all algebraically closed fields of sufficiently large characteristic) and an amusingly slick proof of Ax's theorem (if a polynomial function $\mathbb{C}^n \to \mathbb{C}^n$ is injective, then it is surjective). We will also discuss applications of the Compactness theorem in deriving finitary analogues of the infinitary combinatorial statements such as the infinite Ramsey theorem, van der Waerden's or Szemerédi's theorems, graph colorings, etc.

**Basic recursion theory.** This part will begin with a robust definition of computation (algorithm), followed by a rather short investigation of computable functions and sets. The investigation will be short because we will quickly discover that many interesting functions and sets are not computable, as illustrated by the Gödel Incompleteness Theorem and Church's theorem on undecidability of first-order logic, both of which we will prove.

**And more.** Diving more into model theory, we will study quantifier elimination and model completeness, and, as a quick application, give a transparent proof of Hilbert's Nullstellensatz. If time permits, we will change gears and learn two completely different (set-theoretic and combinatorial) constructions of structures from existing ones: ultraproducts and Fraïssé limits. The former will involve a rather measure-theoretic introduction to ultrafilters, while the latter will touch base with probabilistic objects like the random graph.

TEXTBOOK: Logic lecture notes available on the instructor's webpage [1].

PREREQUISITES: No background in mathematical logic is needed, but knowledge of undergraduate abstract algebra would be helpful.

EXAMS: One in-class midterm and an in-class final.

HOMEWORK: 6–8 problems every week to be submitted in both written and blackboard presentation forms in problem sessions.

[1] http://www.math.uiuc.edu/~anush/