$\boxed{\text{MODEL THEORY}}$

MWF at 12pm	MATH 571	Anush Tserunyan
1066 Lincoln Hall		Fall 2016

Model theory studies mathematical structures that fit well into the framework of first order logic. These include algebraic structures such as groups, rings, fields, modules, as well as combinatorial/set theoretic ones, such as graphs and partial orders. The model-theoretic tools often reveal actually deep and important information about these structures, making model theory a highly applicable subject to other areas of mathematics, especially to algebra, algebraic geometry, algebraic number theory, and combinatorics.

One of the main objects model theory investigates in a given structure is the collection of *definable* sets and functions. Denoting by M the underlying set of a structure $\mathbf{M}$, a set $A \subseteq M^k$ is called *definable* in $\mathbf{M}$ if the property that carves out A from M^k is expressible by the means of first order logic. A function $f : M^k \to M$ is called *definable* if its graph is definable (as a subset of M^{k+1}). We will develop tools for analyzing the definable sets and functions, including *quantifier elimination*, which, when true, allows removing quantifiers from the definitions of the definable sets and functions, limiting what they can be and making them much simpler to analyze. For instance, quantifier elimination for the field of real numbers makes the properties of semialgebraic sets rather transparent, yielding a natural and quick solution of *Hilbert's* 17^{th} *Problem*[1].

An example of a basic tool in model theory is passing to an *elementary extension* of a structure (e.g., its ultrapower), which can be viewed as a completion of the structure, in the same spirit as obtaining the reals from the rationals, but it preserves the combinatorial structure of definable sets and functions.

The course will culminate in the proof of Morley's famous theorem, which is about *categoricity* of a theory T—the phenomenon of any two models of T of the same (large enough) cardinality being isomorphic. For example, vector spaces and algebraically closed fields are categorical and both of these instances hinge on the existence of a basis with respect to a notion of *dependence* (linear for vector spaces, algebraic for fields). The proof of Morley's theorem reveals that whenever the phenomenon of categoricity happens, it is because of the existence of a basis with respect to a certain notion of *dependence*, which is quite remarkable given that the structures under consideration are a priori rather arbitrary.

Prerequisites: Familiarity with basic syntax and semantics of first order logic (the first week of Math 570) will be assumed. Students who haven't taken any course in logic can easily acquire this by reading the first pages of any text in logic, for example, Lou van den Dries's or Anush Tserunyan's lecture notes available on their respective websites.

Required work: Students will be required to write solutions to problems assigned during the course and may also be asked to present them on the board.

Texts and references: There is no required text; Ward Henson's lecture notes will be made available by the instructor. Recent books that can be used as references include *Model Theory: An Introduction* by David Marker and *A Course in Model Theory* by Bruno Poizat.

[1] A rational function over $\mathbb{R}$ is positive semi-definite iff it is a sum of squares of rational functions.