MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

HOMEWORK 9

Due date: Nov 11 (Wed)

Exercises from the textbook. 13.22(a,b), 13.23[1], 13.24, 13.25, 13.26, 13.27, 13.28

Out-of-the-textbook exercises (these are as mandatory as the ones from the textbook)**.**

1. Let (x_n) be a sequence and $x \in \mathbb{R}$. For each of the following conditions, determine whether it implies that x is the limit of (x_n); if YES, then prove it, and if NO, give a counter-example, i.e. give an example of (x_n) and x that satisfy the condition but (x_n) does not converge to x.

(a) $\forall \varepsilon > 0\ \exists K \in \mathbb{N}$ such that $\forall n \geq K$, $x - x_n < \varepsilon$.

(b) $\forall \varepsilon > 0\ \exists K \in \mathbb{N}$ such that $\forall n \geq K$, $x_n \leq x$ and $x - x_n < \varepsilon$.

(c) $x = \sup_{n \in \mathbb{N}} x_n$ and $x = \inf_{n \in \mathbb{N}} x_n$.

(d) $\forall \varepsilon > 0\ \exists K \in \mathbb{N}$ such that $x - \varepsilon < \inf_{n \in \mathbb{N}} x_n \leq \sup_{n \in \mathbb{N}} x_n < x + \varepsilon$.

(e) $\exists K \in \mathbb{N}\ \forall \varepsilon > 0\ \exists n \geq K$, $|x_n - x| < \varepsilon$.

2. Prove that for any sequence $(x_n)_n$ and $L \in \mathbb{R}$, the sequence $(x_n)_n$ converges to L if and only if the sequence $(x_n - L)_n$ converges to 0.

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[1] The hint in the textbook recommends using Proposition 13.15 to show that $\sup A + \sup B$ is the least upper bound for C. I think this is a bit of an overkill and I suggest proving this directly: Fix $u < \sup A + \sup B$ and let ε be the distance between u and $\sup A + \sup B$. Deduce that there are $a \in A$ and $b \in B$ with $\sup A - \frac{\varepsilon}{2} < a$ and $\sup B - \frac{\varepsilon}{2} < b$ and see where this puts $a + b$ in relation to u.