MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

HOMEWORK 8

Due date: Nov 4 (Wed)

Exercises from the textbook. 4.1, 4.2, 13.6, 13.18, 13.19,

Out-of-the-textbook exercises (these are as mandatory as the ones from the textbook)**.**

1. (a) Prove that if a set A is finite and $x \notin A$, then $|A \cup \{x\}| = |A| + 1$.
   (b) Deduce that if a set B is finite and $x \in B$, then $|B - \{x\}| = |B| - 1$.
   (c) Conclude that if a set B is infinite, then for any element x (not necessarily in B), the set $B - \{x\}$ is still infinite.

2. Prove that every finite nonempty set $A \subseteq \mathbb{R}$ has a maximum and a minimum elements.
   HINT: Induction on $n := |A|$, using Problem 1b in the step of induction.

3. Determine the 3-ary and 4-ary expansions of $2835_{(10)}$.

4. Let $A, B \subseteq \mathbb{R}$ be nonempty sets bounded above. Prove that $\sup(A \cup B) = \sup\{\sup A, \sup B\}$.

5. For each set S below, find $\inf S$ and $\sup S$, and prove your answers.
   (a) $S = \left\{1/\sqrt{n} : n \in \mathbb{N}\right\}$;
   (b) $S = \{1/n - 1/m : n, m \in \mathbb{N}\}$.

6. Let (x_n) be a sequence and $x \in \mathbb{R}$. For each of the following conditions, determine whether it implies that x is the limit of (x_n); if YES, then prove it, and if NO, give a counter-example, i.e. give an example of (x_n) and x that satisfy the condition but (x_n) does not converge to x.
   (a) $\forall \varepsilon > 0\ \exists N \in \mathbb{N}$ such that $\forall n \geq N$, $|x - x_n| < \varepsilon$.
   (b) $\forall \varepsilon > 0\ \exists N \in \mathbb{N}$ such that $\forall n \geq N$, $|x - x_N| < \varepsilon$.
   (c) $\forall \varepsilon > 0\ \exists N \in \mathbb{N}$ such that $|x - x_N| < \varepsilon$.
   (d) $x = \sup\{x_n : n \in \mathbb{N}\}$ or $x = \inf\{x_n : n \in \mathbb{N}\}$.
   (e) $\forall K \in \mathbb{N}\ \exists \varepsilon > 0$ such that $\forall n \geq K$, $|x_n - x| < \varepsilon$.
   (f) $\exists K \in \mathbb{N}\ \forall \varepsilon > 0\ \forall n \geq K$, $|x_n - x| < \varepsilon$.