DESCRIPTIVE SET THEORY

HOMEWORK 1

Due on Tuesday, Feb 4

1. Let x, y, A, B be sets.
   (a) Show that $\{x\}$ is a set.
   (b) Show that $(x, y) := \{\{x\}, \{x, y\}\}$ is a set and write a formula $\phi(z)$ that holds if and only if z is an ordered pair. Moreover, write formulas $\phi_0(z, x)$ and $\phi_1(z, y)$ such that $\phi_0(z, x)$ and $\phi_1(z, y)$ hold if and only if $z = (x, y)$. In other words, ϕ_0 defines the function $z \mapsto x$ and ϕ_1 defines the function $z \mapsto y$.
   (c) Show that $A \times B := \{(x, y) : x \in A \wedge y \in B\}$ is a set.
   (d) Define the notion of a function $f : A \to B$ as a certain subset of $A \times B$, i.e. write down which sets are called functions from A to B.

2. (Present) Finish the proof of Lemma 2.6.

3. (Present) Show that the powerset of a transitive set is transitive.[1]

4. (Present) Prove part (c) of Lemma 3.3 and part (a) of Lemma 3.4. Do not use any of the later parts of Lemmas 3.4 and 3.5 in your proofs.

5. Prove that there does NOT exist a set that contains all of the ordinals.

6. Prove Lemma 3.11.

7. (Present) Prove Proposition 5.4.

8. (Present) Prove that $\mathbb{N} \equiv \mathbb{Z} \equiv \mathbb{Q}$.

9. (Present) Prove that $\mathbb{R} \equiv (0, 1) \equiv [0, 1] \equiv 2^{\mathbb{N}} \equiv \mathrm{Pow}(\mathbb{N})$.

[1] Thanks to Travis Nell for suggesting this exercise.