Math 570: MATHEMATICAL LOGIC

Fall 2014, MWF 3:00pm-3:50pm in 445 Altgeld Hall

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At the beginning of the 20^{th} century, mathematics experienced a crisis due to the discovery of certain paradoxes (e.g. Russell's paradox) in previous attempts to formalize abstract notions of sets and functions. To put analysis on a firm foundation, similar to the axiomatic foundation for geometry, Hilbert proposed a program aimed at a direct consistency proof of analysis. This would involve a system of axioms that is consistent, meaning free of internal contradictions, and complete, meaning rich enough to prove all true statements. But the search for such a system was doomed to fail: Gödel proved in the early 1930s that any system of axioms that can be listed by some computable process and subsumes Peano arithmetic, is either incomplete or inconsistent. This is the Gödel Incompleteness Theorem, and we will prove it in the second half of this course after developing some recursion-theoretic background.

In the first half, we will develop the framework of first order logic, culminating in proofs of Gödel's Completeness Theorem and the Compactness Theorem, one of the most useful tools of logic. We will further dive into model theory, exploring categoricity, quantifying elimination, model completeness, and more.

Throughout the course, we will give applications in various fields of mathematics such as combinatorics and algebra. In particular, we will consider the theory of algebraically closed fields and derive the Lefschetz Principle (a first-order sentence is true in the field of complex numbers if and only if it is true in all algebraically closed fields of sufficiently large characteristic), as well as Hilbert's Nullstellensatz.

TEXTBOOK: none needed, we will use lecture notes that will be posted on the course webpage.

PREREQUISITES: no background in mathematical logic is needed, but knowledge of undergraduate abstract algebra would be extremely helpful.

EXAMS: one in-class midterm and an in-class final.

HOMEWORK: 8-9 problems every week; these will either be turned in to the TA for grading or presented in problem sessions.

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