

quiz 3

math228, classical geometry

fall 2021

The quiz is worth 10 points. Justify all your claims rigourously.

Recall that for any triangle ABC , the following holds :

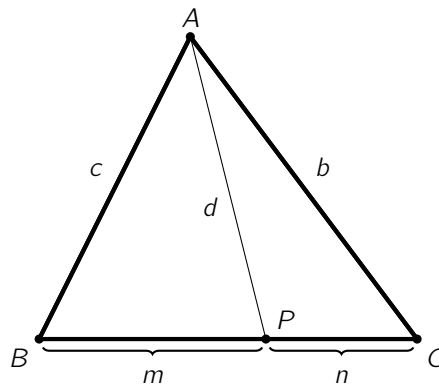
$$|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|\cos(\angle BAC)$$

This previous equation is often referred to as the *cosine law*.

(10 points) **a.** Consider a triangle ABC and a point P on the segment BC such that

$$|BP| = m, \quad |CP| = n, \quad |AP| = d, \quad |AC| = b, \quad |AB| = c,$$

as pictured in the following figure :



Using the cosine law and the fact that $\cos(180^\circ - \alpha) = -\cos \alpha$ for any angle α , show that

$$b^2m + c^2n = (m+n)(d^2 + mn).$$

This result is known as *Stewart's theorem*.

(5 points) **b.** By applying **a**, show that for any triangle ABC having centroid G ,
bonus

$$|AB|^2 + |AC|^2 + |BC|^2 = 3(|AG|^2 + |BG|^2 + |CG|^2).$$