

# midterm 1

## math228, classical geometry

### fall 2021

The midterm 1 is worth 25 points. Justify all your claims rigourously.

#### state (5 points)

In this section, you must answer both questions.

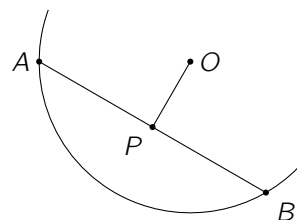
1. State Euclid's 5 postulates.
2. State the Pythagorean theorem.

#### straight edge-compass constructions (10 points)

In this section, choose one question to answer.

3. Consider a circle with center  $O$  and two points  $A, B$  lying on the given circle. Let  $P$  be a point on the chord  $AB$  such that  $AB$  is perpendicular to  $OP$ .

- a. Show that  $P$  bisects  $AB$ .
- b. Give a straight edge-compass construction to find the center of a circle.

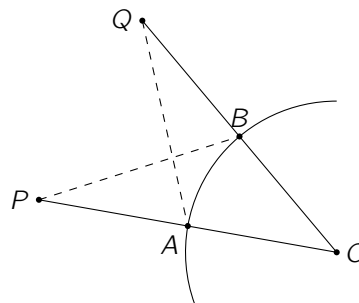


4. Consider a circle with center  $O$  and a point  $P$  lying outside the given circle. Let  $A$  be the intersection point of the segment  $OP$  with the circle.

- a. Let  $Q$  be point different from  $P$  lying outside the circle such that  $|OP| = |OQ|$ . Let  $B$  denotes the intersection point of  $OQ$  with the given circle. Show that

$$\angle OBP = \angle OAQ.$$

- b. Give a straight edge-compass construction to find a tangent line to a circle passing through a given point outside the circle.



### elementary geometry (10 points)

In this section, choose one question to answer.

5. Let  $ABC$  be a triangle, where

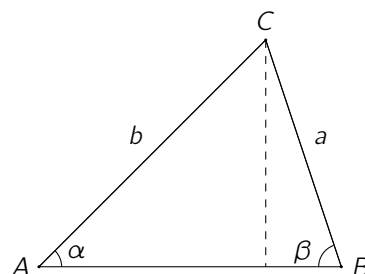
$$|AB| = c, |AC| = b, |BC| = a, \\ \angle BAC = \alpha, \angle ABC = \beta, \angle ACB = \gamma.$$

a. By dropping a height from  $C$  to  $AB$ , show that

$$c^2 = ac \cos \beta + bc \cos \alpha \\ = c(a \cos \beta + b \cos \alpha).$$

b. Deducing analogous formulas as in a for the 2 other edges, conclude that

$$a^2 + b^2 - 2ab \cos \gamma = c^2.$$



6. Let  $AB$  be a diameter of a circle. Show that if  $C$  is a point on the circle different from  $A$  and  $B$ , then  $\angle ACB$  is a right angle.

