## quiz 1 math222, calculus III summer 2025

Justify all your claims rigorously. Allotted time is 1 hour.

**1.** Determine whether the following series are convergent or divergent. Specify the convergence criteria you are applying.

**a.** 
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$$
  
**b.**  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n) \ln(\ln(n))}$   
**c.**  $\sum_{n=1}^{\infty} \frac{1}{(3 + (-1)^n)^n}$ 

**2.** The hyperbolic functions "sinh" and "cosh" are the unique functions such that sinh(0) = 0, cosh(0) = 1 and which satisfy the differential equations

$$\frac{d}{dx}\cosh(x) = \sinh(x)$$
 and  $\frac{d}{dx}\sinh(x) = \cosh(x)$ 

for all  $x \in \mathbb{R}$ .

- **a.** Compute the Taylor series of sinh and cosh centered at a = 0.
- **b.** Assuming that the series computed in **a** coincide with their corresponding function, prove that the following identity holds :

$$\left(\cosh(x)\right)^2 - \left(\sinh(x)\right)^2 = 1.$$

*Hint* : Consider the expressions  $\cosh(x) - \sinh(x)$  and  $\cosh(x) + \sinh(x)$ .