

MATH 340: Discrete Structures II. Winter 2016.
Due in class on Tuesday, April 12th.

Assignment #6: Generating functions.

1. *Fruit salad.* Let $s(n)$ be the number of ways to make a fruit salad with n pieces of fruit, given that we must use strawberries by the half-dozen, an odd number of apples, between 2 and 7 bananas, and at most one pineapple.

a) Evaluate the ordinary generating function for s .

b) Use this to find $s(n)$.

2. *The Round table.* Let $r(n)$ be the number of different ways to seat n people around a round table. Find the exponential generating function for r .

3. *Sum of cubes.*

Use generating functions to evaluate

$$\sum_{k=0}^n k^3$$

4. *Alternating Permutations.* A permutation $\pi_1, \pi_2, \dots, \pi_n$ of numbers $1, 2, \dots, n$ is *alternating* if

$$\pi_1 > \pi_2 < \pi_3 > \pi_4 < \dots$$

Let $a(n)$ be the number of alternating permutations of size n .

a) Find a recurrence relation for $a(n)$.

b) Evaluate the exponential generating function for a .