MATH 340: Discrete Structures II. Winter 2016.
Due in class on Tuesday, March 8th.

Assignment #3: Discrete Probability.

1. *Bayes Theorem.*

a) *Cards.* Alice has three cards: one red on both sides, one black on both sides, and one red on one side and black on the other. She mixes them up in a bag, draws one at random, and places it on the table with a red side showing. What is the probability that the other side is also red?

b) *A die and a coin.* Bob rolls a single six-sided die, and then flips a coin the number of times showing on the die. The coin comes up heads every time. What is the probability that the die showed 6?

c) *Drug test.* A large company gives a new employee a drug test. The False-Positive rate is 1% and the False-Negative rate is 5%. In addition, 1% of the population use the drug. The employee tests positive for the drug. What is the probability the employee uses the drug?

2. *Monty Hall variant.* As in the original problem, the car is equally likely to be behind either one of the three doors, numbered #1, #2 and #3. However, after you selected a door the host opens the door with the lowest number among the doors that you did not select and that don't contain a car. (For example, if you selected Door #1 and the car was behind this door, then the host would always open Door #2, and never Door #3.) Suppose that you select Door #1.

a) Suppose further that the host opens Door #2. What are the probabilities that you win the car if you stick to your choice, and if you switch?

b) What if the host opens Door #3?

3. *Independence and sampling.* Let A and B be events such that $0 < p(A) < 1, 0 < p(B) < 1$. Suppose that

$$p(A \,|A \cup B) p(B \,|A \cup B) = p(A \cap B \,|A \cup B).$$

Are the events A and B independent, positively or negatively correlated? Justify your answer.

4. *Markov inequality.* Let X be a random variable taking only non-negative values, and let c be a positive constant. Show that

$$p(X \geq c) \leq \mathrm{E}(X)/c.$$

5. *Binomial distribution.* The Stanley Cup winner is determined in the final series between two teams. The first team to win 4 games wins the Cup. Suppose that Montréal Canadiens advance to the final series, and they have a probability of 0.6 to win each game, and the game results are independent of each other. Find the probability that

a) Canadiens win the Stanley cup.

b) Seven games are required to determine the winner

6. *Geometric distribution.* Suppose we run repeated independent Bernoulli trials (with success probability p) until we obtain a success. Let the random variable X be the number of trials needed before we obtain a success.

a) Calculate the probability $P(X = k)$ for an integer k.

b) Prove that $E(X) = 1/p$.