MATH 340: Discrete Structures II. Winter 2016.
Due in class on Tuesday, February 16th.

Assignment #2: Planar graphs.

1. *Euler's formula.*

a) Let G be a planar graph, such that every vertex of G has degree at least five, and at least one vertex of G has degree ten. Show that G has at least seventeen vertices.

b) Let G be a graph drawn in the plane. Suppose that every face of G is bounded by a cycle of odd length. Show that the number of faces of G is even.

2. *Coloring planar graphs.*

a) Show without using the Four Color Theorem that if a planar graph G has no K_3 subgraph then $\chi(G) \leq 4$.

b) Prove or disprove the following statement: If a planar graph G has no K_4 subgraph then $\chi(G) \leq 3$.

Hint: In a) show that G contains a vertex of degree at most three.

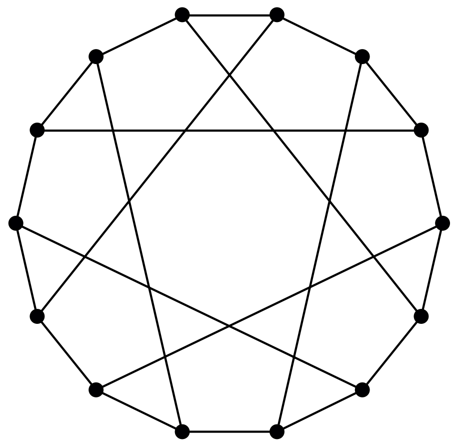
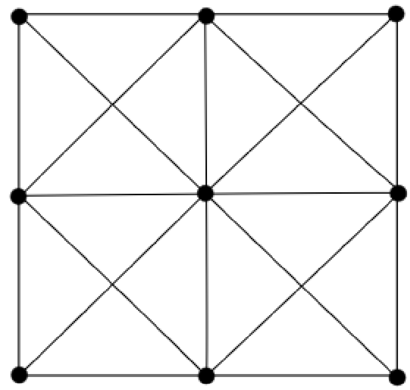
3. *Art Gallery theorem.* Prove or disprove the following statements.

a) If a gallery can be guarded by one guard then it is convex.

b) If a gallery can not be guarded by one guard then it has at least six walls.

c) If a gallery has at least six walls then it can not be guarded by one guard.

4. *Kuratowski's theorem.* Let G be a connected non-planar graph with m edges and n vertices. Suppose further that $G \setminus e$ is planar for every edge e of G. Show that $m - n = 3$, or $m - n = 5$.

5. *Testing planarity.* Determine whether the above two graphs are planar. (For each graph either provide a planar drawing, or prove that this graph is not planar.)