Classifying bundles with fiber $K(\pi, 1)$

In the Postnikov system for a map $f : X \to B$ there are commutative diagrams

$$\begin{tikzcd}
 & X^n \arrow[dd, "q^n"] \arrow[dr, "p^n"] & \\
X \arrow[ur, "u^n"] \arrow[dr, "u^{n-1}"'] & & B \\
 & X^{n-1} \arrow[ur, "p^{n-1}"'] &
\end{tikzcd}$$

where the u^n are n-connected and the p^n and q^n n are n-covers for $n \geq 0$. If F is the fiber of f and F^n the fiber of p^n, then we have diagrams

$$\begin{tikzcd}
F \arrow[rr] \arrow[dr] & & F^n \arrow[dl] \\
 & F^{n-1} &
\end{tikzcd}$$

which are a Postnikov system for F. Looking at the commutative diagrams above, we see that the fiber of q^n is the fiber of $F^n \to F^{n-1}$ which is $K(\pi_n(F), n)$. Assuming f is a minimal fibration, the q^n will also be minimal fibrations and hence bundles with fiber $K(\pi_n(F), n)$. So, for $n \geq 2$, The situation is as we indicated at CT 2011. The classifying spaces are homotopy colimits of diagrams of $K(\pi_n, n+1)$'s over the fundamental groupoid.

The case $n = 1$, is different, however, for then the fiber is $K(\pi_1(F), 1)$, which, in general, is not a simplicial abelian group etc. If we assume F is connected, then $X^0 = B$ and $q^1 : X^1 \to B$ is a bundle with fiber $K(\pi_1(F), 1)$, so we need to know how to classify these. In the literature this problem is either ignored, leading to mistakes, or swept under the rug by assuming $\pi_1(F)$ is abelian. In the talk, we give a complete, general solution to the problem. This is joint work with Andr Joyal.