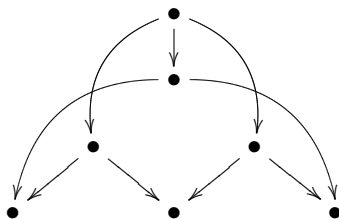


Span refinements of the Cech Nerve

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A 2-span is a commutative diagram of the form



Given a cover (epimorphism) $U \rightarrow 1$ in a topos $\mathcal{E} \xrightarrow{\gamma} \mathcal{S}et$, it determines a simplicial object U_\bullet , $U_n = U \times U \times \dots \times U$ $n + 1$ times. A hypercover X_\bullet is a simplicial object together with a morphism $X_\bullet \rightarrow U_\bullet$. If the topos is locally connected simplicial objects have a canonical indexing simplicial set $\gamma!(X_\bullet)$, $X_\bullet \rightarrow \gamma^*\gamma!(X_\bullet)$. For non locally connected topoi the indexing has to be given explicitly as part of the datum. This leads to the notion of *simplicial family*, $X_\bullet \rightarrow \gamma^*(S_\bullet)$ (where S_\bullet is a simplicial set) and of *indexed hypercover*. We discover that the n -simplexes of a simplicial family furnish a notion of n -spans which is intimately related to the coeskeleton functor. We develop this theory and an application to the fundamental progroupoid of a non locally connected topos. In this talk we will not assume familiarity with hypercovers or the coeskeleton functor.