Title: 2-exactness: a study in 2-dimensional categorical logic.

Abstract: In category theory, we often introduce a desired new (small) category *A*, possibly on the basis of already given categories, functors, etc., by declaring what **Ob(***A*), the *set* of objects of the new category is to be, then what **Arr**(*A*), the *set* of arrows is to be, and proceed naturally afterward. Think of the example of the opposite of a given category. I propose to internalize the definition of *A* in **Cat**, the 2-category of small categories. We do not have direct reference to sets (for instance, the set of objects of a given category). I assume that we have categories *C_0* and *C_1*, i.e., objects of **Cat**, and we have the requirements that **Ob**(*A*) be **Ob**(*C_0*), and **Arr**(*A*) be **Ob**(*C_1*). It turns out – and this is the main point of the talk to be given – that a certain *natural* further structure on the pair (*C_0*,*C_ 1*), called *2-equivalence*, formulated purely in terms of the 2-category **Cat** with its finite 2-limit structure, enables us to complete the construction of the category *A* with suitable connections to *C_0* and *C_1*, in particular a functor *g: C_0------>A*. Conceptually, *g: C_0------>A* will be the *quotient* of the 2-equivalence.

The concepts mentioned above give rise to the notion of *2-exact 2-category.* 2-exactness of 2-categories have been considered by Ross Street and others. I still have to find the precise connections between my notions to the ones in the literature; it is quite possible that the notions I introduce are identical to ones already published. The point of the present study is the use of 2-exactness in formulating internally in a 2-category definitions of a “category” such as the opposite of a given category, and, as a more involved example, the category of pullback squares with monomorphisms as verticals of a given category with pullbacks. Readers will recognize that the operation of taking the underlying groupoid of a “category”, a right-adjoint-type 2-categorical operation, will be indispensable as a prerequisite, for instance for the definition of the “opposite”. The present study is considered part of a program of defining of a suitable concept of “2-topos”, related to but not identical to the same-named concept introduced by Mark Weber. Eventually, the goal is to internalize 2-topos-theory in the non-classical logic of a Gray-category.