Definition. Recall that a semiprime ring is DL-closed if given any elements r, s s.t. $r^3 = s^2$, then there is a unique t s.t. $t^2 = r$ and $t^3 = s$.

The 15th condition mentioned in the slides is simply that the canonical map $K(D) \longrightarrow G(D)$, whose existence is forced by adjunction, is injective. Somebody asked whether that was always injective and this is the answer. It is trivially implied by 8 and equally trivially implies 5. Despite its triviality, it gives useful insight into rougeosity.

Here is a proof of the fact that if a prime contains an intersection of a compact set of primes in the domain topology, it contains at least one of them. Suppose U is such a set and that $\bigcap_{P \in U} \subseteq Q$. If $P \not\subseteq Q$, then there is an element $r_P \in P - Q$. The sets $Z(r_P)$ cover U and so there are $P_1, \ldots, P_n \in U$ s.t. $\{Z(r_{P_i})\}$ covers U. Let $r = \prod r_{P_i}$. Then it is clear that $r \in P$ for all $P \in U$, while $r \notin Q$.

Here is how you build a local representation. Assuming $\zeta(P) = r \in R$, it will equal r on an open set U_P containing P. It will do so on a basic open set containing P and the basic open sets are finite meets of sets of the form $Z(r)$ which are clopen in the patch topology, hence compact in that any weaker topology. Thus we can assume that U_P is compact and open. Then U can be covered by finitely many, etc.

Here is the statment of the "main theorem" whose proof will be sketched in this talk:

0.1. Theorem. *Let R be a commutative semiprime ring. Then the following are equivalent:*

DL-1. *R is DL-closed.*

DL-2. *R is isomorphic, under the canonical map, to the ring of global sections of the sheaf E_R.*

DL-3. *R is isomorphic to a ring of global sections of sheaf whose stalks are domains.*

DL-4. *R is in the limit closure of the domains.*

Three topologies: The domain topology has as subbase sets $Z(r) = \{P \mid r \in P\}$. The Zariski topology has as (sub)base sets $N(r) = \{P \mid r \notin P\}$. The patch topology takes all the sets $N(r)$ and $Z(r)$ as subbase. The last is compact, Hausdorff, and totally disconnected. It follows that all sets of the form $N(r)$ and $Z(r)$ are compact in all three topologies.

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