```
> ## Assignment #9 Mast 334/Math 354 Solutions
> ## Problems 1e and 2e page 506:
> f:=x->cos(x)/2+sin(2*x)/3; ## x in [0,1]
# Linear least squares :  P1(x)=ax+b
# Equations:
EQ1:= a*int(x^2,x=0..1.0) +
b*int(x,x=0..1.0)=int(x*f(x),x=0..1.0);
EQ2:= a*int(x,x=0..1.0) +
b*int(1,x=0..1.0)=int(f(x),x=0..1.0);
```

$$f := x \to \frac{1}{2}\cos(x) + \frac{1}{3}\sin(2\,x)$$

$$EQ1 := 0.3333333333\ a + 0.5000000000\ b = 0.3360192370$$

$$EQ2 := 0.5000000000\ a + 1.0\ b = 0.6567599652$$

```
> solve({EQ1,EQ2},{a,b});
```

$$\{\, b = 0.6109244388\,,\ a = 0.09167105284 \,\}$$

```
> ## Answer: P1(x)=0.091671*x+0.610924
##_____________________________________________________________
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##
# Quadratic least squares :  P2(x)=ax^2+bx+c
# Equations:
a:='a':b:='b':c:='c':
EQ1:= a*int(x^4,x=0..1.0) + b*int(x^3,x=0..1.0) +
c*int(x^2,x=0..1.0)=int(x^2*f(x),x=0..1.0);
EQ2:= a*int(x^3,x=0..1.0) + b*int(x^2,x=0..1.0)+
c*int(x,x=0..1.0)=int(x*f(x),x=0..1.0);
EQ3:= a*int(x^2,x=0..1.0) + b*int(x,x=0..1.0)+
c*int(1,x=0..1.0)=int(f(x),x=0..1.0);
```

$$EQ1 := 0.2000000000\,a + 0.2500000000\,b + 0.3333333333\,c = 0.2224619543$$

$$EQ2 := 0.2500000000\,a + 0.3333333333\,b + 0.5000000000\,c = 0.3360192370$$

$$EQ3 := 0.3333333333\,a + 0.5000000000\,b + 1.0\,c = 0.6567599652$$

```
> solve({EQ1,EQ2,EQ3},{a,b,c});
```

$$\{\, a = -0.7375119328\,,\ c = 0.4880057831\,,\ b = 0.8291829861 \,\}$$

```
> ## Answer: P2(x)=-0.737511*x^2+0.829182*x+0.488006
>
> ## Problem 11 page 506
> ## Laguerre polynomials on [0,infinity) with w(x)=exp(-x)
L0:=x->1;
```

$$L0 := 1$$

```
> w:=x->exp(-x);
B1:=int(x*w(x),x=0.0..infinity)/int(w(x),x=0.0..infinity);
```

$$w := x \to \mathbf{e}^{(-x)}$$

$$B1 := 1.000000000$$

```
> L1:=x->x-1;
## Just to check we calculate
int(L0(x)*L1(x)*w(x),x=0.0..infinity);
```

$$L1 := x \to x - 1$$

$$-0.4857225733\ 10^{-16}$$

```
> ## It is practically 0
B2:=int(x*w(x)*L1(x)^2,x=0.0..infinity)/int(w(x)*L1(x)^2,x=
0.0..infinity);
C2:=int(x*w(x)*L1(x)*L0(x),x=0.0..infinity)/int(w(x)*L0(x)^
2,x=0.0..infinity);
```

$$B2 := 3.00000000$$

$$C2 := 1.00000000$$

```
> L2:=x->(x-3)*L1(x)-1*L0(x);expand(L2(x));
```

$$L2 := x \to (x-3)\,\mathrm{L1}(x) - \mathrm{L0}(x)$$

$$x^2 - 4\,x + 2$$

```
> ## Just to check we calculate
int(L0(x)*L2(x)*w(x),x=0.0..infinity);
int(L1(x)*L2(x)*w(x),x=0.0..infinity);
```

$$0.260208538\,10^{-16}$$

$$0.428259858\,10^{-16}$$

```
>
B3:=int(x*w(x)*L2(x)^2,x=0.0..infinity)/int(w(x)*L2(x)^2,x=
0.0..infinity);
C3:=int(x*w(x)*L2(x)*L1(x),x=0.0..infinity)/int(w(x)*L1(x)^
2,x=0.0..infinity);
```

$$B3 := 5.000000000$$

$$C3 := 4.000000000$$

```
> L3:=x->(x-5)*L2(x)-4*L1(x);expand(L3(x));
```

$$L3 := x \to (x-5)\,\mathrm{L2}(x) - 4\,\mathrm{L1}(x)$$

$$x^3 - 9\,x^2 + 18\,x - 6$$

```
> ## Just to check we calculate
int(L0(x)*L3(x)*w(x),x=0.0..infinity);
int(L1(x)*L3(x)*w(x),x=0.0..infinity);
int(L2(x)*L3(x)*w(x),x=0.0..infinity);
```

$$0.1419221632\ 10^{-15}$$

$$0.7315654159\ 10^{-16}$$

$$-0.5697827582\ 10^{-15}$$

```
> ### Problem 12b page 507
> f:=x->exp(-x);
```

$$f := x \rightarrow \mathbf{e}^{(-x)}$$

```
> ## Calculating coefficients:
a0:=int(f(x)*L0(x)*w(x),x=0.0..infinity)/int(w(x)*L0(x)^2,x
=0.0..infinity);
a1:=int(f(x)*L1(x)*w(x),x=0.0..infinity)/int(w(x)*L1(x)^2,x
=0.0..infinity);
a2:=int(f(x)*L2(x)*w(x),x=0.0..infinity)/int(w(x)*L2(x)^2,x
=0.0..infinity);
a3:=int(f(x)*L3(x)*w(x),x=0.0..infinity)/int(w(x)*L3(x)^2,x
=0.0..infinity);
```

$$a0 := 0.5000000000$$

$$a1 := -0.2500000000$$

$$a2 := 0.06250000000$$

$$a3 := -0.01041666667$$

```
> ## Linear:
P1:=x->a1*L1(x)+a0*L0(x):expand(P1(x));
```

$$-0.2500000000\ x + 0.7500000000$$

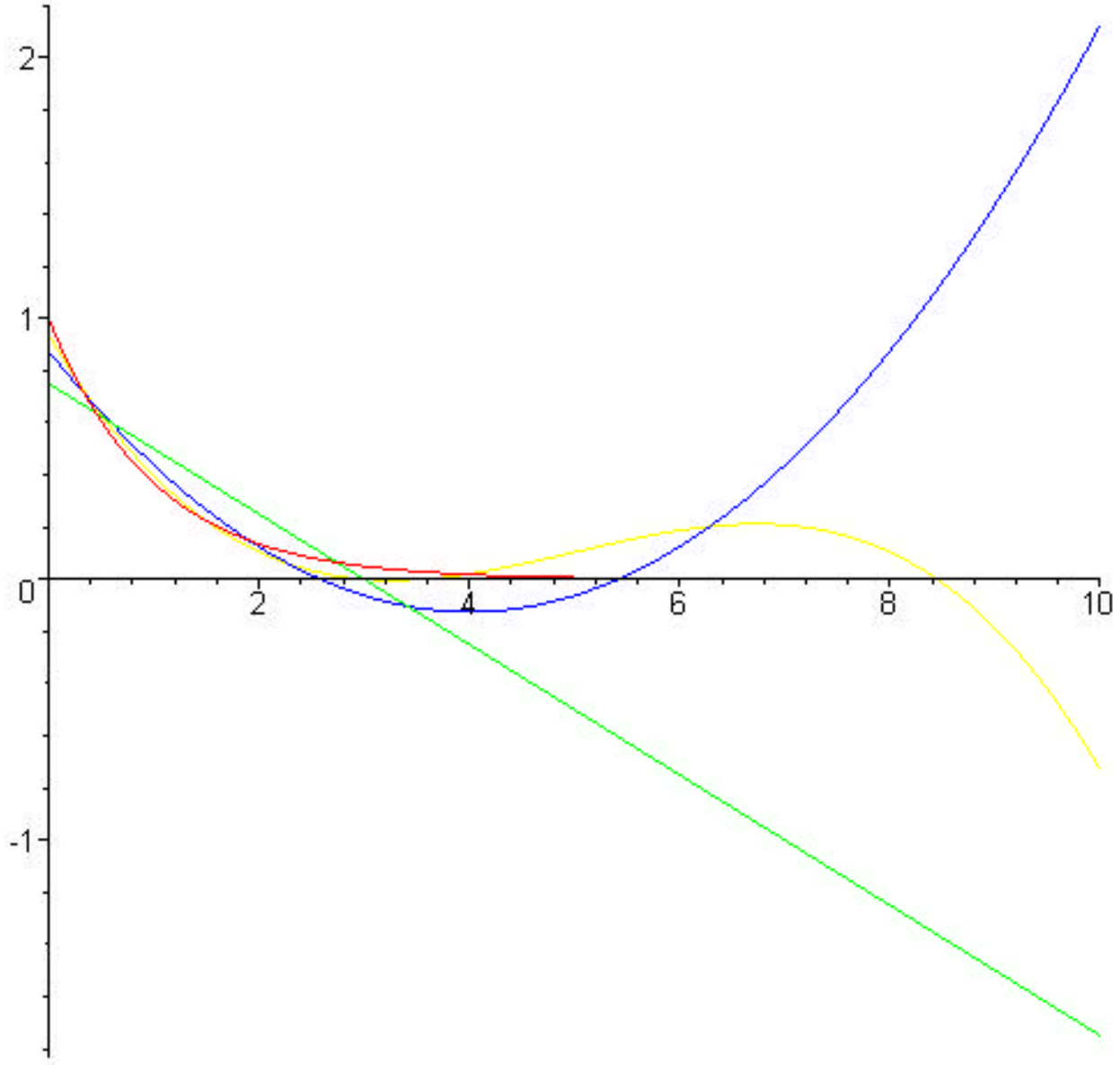
```
> ## Quadratic:
P2:=x->P1(x)+a2*L2(x):expand(P2(x));
```

$$-0.5000000000\ x + 0.8750000000 + 0.06250000000\ x^2$$

```
> ## Cubic:
P3:=x->P2(x)+a3*L3(x):expand(P3(x));
```

$$-0.6875000000\ x + 0.9375000000 + 0.1562500000\ x^2 - 0.01041666667\ x^3$$

```
> plot([f,P1,P2,P3],0..10,color=[red,green,blue,yellow]);
```


2
1
0
-1
2
4
6
8
10

>