

**CONCORDIA UNIVERSITY**  
**Department of Mathematics & Statistics**

Course	Number	Section(s)
Mathematics	203	All
Examination	Date	Pages
Final	April 2007	3
Instructors	Course Examiner	
B. Brown, J. Hayes, R. Mearns, M. Mei, D. Sevilla-Gonzalez	H. Proppe	
Special Instructions		
▷ Calculators are <b>not</b> allowed.		

MARKS

- [8] 1. (a) Let  $f(x) = x^2 + 1$  and  $g(x) = \sin x$ . Find  $f \circ g$  and  $g \circ f$ .  
 (b) Find the inverse of the function  $f(x) = e^{2x} - 1$ . Determine the domain and range of  $f$  and  $f^{-1}$ .

- [10] 2. Evaluate the limits:

$$(a) \lim_{x \rightarrow 7} \frac{x^2 - 6x - 7}{\sqrt{x+2} - 3} \quad (b) \lim_{x \rightarrow -\infty} \frac{4x^2 \sqrt{9x^6 + 2x^2}}{x^5 + 3}$$

Do not use l'Hopital's rule.

- [12] 3. (a) Consider the function  $f(x) = \frac{|x+1|}{x^2-1}$ .

Calculate both one-sided limits at the point(s) where the function is undefined.

- (b) Find parameters  $a$  and  $b$  such that the function

$$f(x) = \begin{cases} 1 - \frac{x^2 - x}{x^2 - 1}, & \text{if } 0 \leq x < 1 \\ a, & \text{if } x = 1 \\ x + b, & \text{if } x > 1 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

[12] 4. Find derivatives of the functions (do not simplify the answer):

(a)  $f(x) = e^{-x^2} (\cos x + \tan x)^2$  ;

(b)  $f(x) = \frac{\ln(1+x^2)}{1+\arctan(2x)}$  ;

(c)  $f(x) = \sec^3(\sqrt{1+x^2})$  ;

(d)  $f(x) = (\sin(5x))^{x^3}$  (use logarithmic differentiation).

[12] 5. Given the function  $f(x) = \sqrt{3x+1}$ ,

(a) Use appropriate differentiation rules to find the derivative of the function.

(b) Use the definition of derivative to verify (a).

(c) Find the differential of the function at  $x = 1$ .

(d) Use the differential above with the appropriate choice of  $dx = \Delta x$  to estimate  $\sqrt{5}$ .

[10] 6. (a) A curve called a lemniscate is defined implicitly by the equation  $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ . Verify that the point  $(3, 1)$  belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Use l'Hopital's rule to evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{(\ln(x+1))^2}$ .

[8] 7. (a) Let  $f(x) = \arcsin(x^2)$ . Find  $f''(x)$ .

(b) Let  $f(x) = 3x^2 + 2x + 5$ . Find a number  $c$  that satisfies the Mean Value Theorem for the function  $f(x)$  on  $[-1, 1]$ .

- [12] 8. (a) A particle is moving along the curve  $y = 4\sqrt{x}$ . As the particle passes the point  $(9, 12)$  its  $x$ -coordinate is increasing at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant?
- (b) Find the area of the largest rectangle that has its base on the  $x$  - axis and its other two vertices above the  $x$  - axis and lying on the parabola  $y = 12 - x^2$ .
- [16] 9. Given the function  $f(x) = \frac{e^x}{x}$ ,
- (a) Find the domain and check for symmetry. Find asymptotes (if any).
- (b) Calculate  $f'(x)$  and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).
- (c) Calculate  $f''(x)$  and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).
- (d) Sketch the graph of the function.

[5] **Bonus Question**

Let  $f(x) = x|x|$ . Use the definition of the derivative to show that  $f'(0)$  exists, then find  $f'(x)$  for any  $x$ .