Champlain College – St.-Lambert

MATH 201: Calculus I

Review for Final Examination

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Questions

1. (24pts) Find the following limits if they exist. If they do not exist, briefly indicate why not and determine they go to $-\infty$, ∞, or neither.

    (a) $\lim\limits_{x \to 2} \frac{x-2}{3x^2-12}$;

    (b) $\lim\limits_{x \to \infty} \frac{\sqrt{4x-3}}{2x+3}$;

    (c) $\lim\limits_{x \to 1} \left(\frac{1}{1-x} - \frac{1}{x-x^2} \right)$;

    (d) $\lim\limits_{x \to 3} \frac{|x-3|}{x-3}$;

2. (24pts) Find the derivative for each of the following functions.

    (a) $f(x) = 2x^2 - 3\sqrt[3]{x} + \frac{1}{x^2}$;

    (b) $f(x) = (x^3 + \pi)e^{-x}$;

    (e) $f(x) = \frac{x^3-1}{x \ln x}$;

    (f) $f(x) = e^{x^2}(x-1)$.

3. (6pts) Let $f(x) = \frac{x-2}{x^2-x-2}$. Find the value of x where this function is discontinuous. State the type of discontinuity as removable, jump or infinity.

5. (6pts) Find the equation of the tangent line to the graph $2x^2 - xy - 8 = 0$ at the point $(2,0)$.

6. (6pts) Let

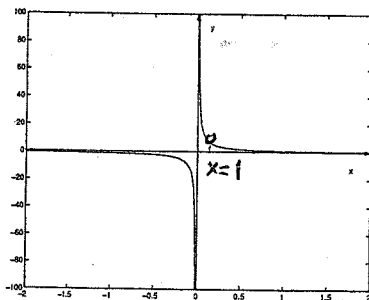$$f(x) = \begin{cases} x^2 - 1, & x \le 0 \\ 2x - 1, & x > 0. \end{cases}$$

Figure 0.1: The graphs of $f(x) = \frac{x-1}{x^2-x}$

Short Answers

Q1. (a). $\lim\limits_{x\to 2} \frac{x-2}{3x^2-12} = \frac{1}{12}$.   (b). $\lim\limits_{x\to\infty} \frac{\sqrt{4x-3}}{2x+3} = 0$.   (c). $\lim\limits_{x\to 1}\left(\frac{1}{1-x} - \frac{1}{x-x^2}\right) = -1$.
(d). $\lim\limits_{x\to 3} \frac{|x-3|}{x-3}$ doesn't exist, because the left limit is -1 and the right limit is 1, they are not equal.

Q2. (a). $f'(x) = 4x - x^{-\frac{2}{3}} - \frac{2}{x^3}$.   (b). $f'(x) = (3x^2 - x^3 - \pi)e^{-x}$.

(e). $f'(x) = \frac{(2x^3+1)\ln x - x^3 + 1}{x^2(\ln x)^2}$.

(f). $f'(x) = e^{x^2}(2x^2 - 2x + 1)$.

Q3. $x = 2$ is the removable-discontinuous point, and $x = -1$ is the infinity-discontinuous point.

Q5. the equation of the tangent line at $(2,0)$ is $y = 4x - 8$.

Q6. $x = 0$ is not differentiable, because it is a sharp point.
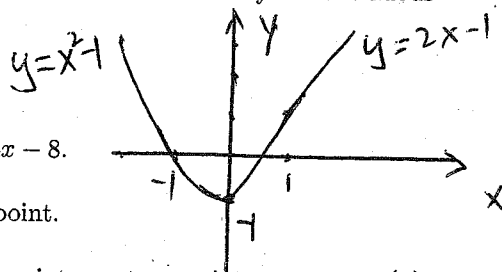
Q7. (a). $D = (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.   (b). No x-intercept, no y-intercept.   (c). Vertical asymptote: $x = 0$ and the horizontal asymptote is $y = 0$.   (d). $f(x)$ is increasing in $(-\infty, 0)$, $(0, 1)$ and $(1, \infty)$, respectively. No critical number, no local maximum, no local minimum.   (e). $f(x)$ is concave upward in $(0,1)\cup(1,\infty)$ and concave downward in $(-\infty, 0)$, no inflection points.   (f). No absolute maximum, nor absolute minimum. The graph of the function is shown in Figure 0.1.

Q8. (a). Let x be the number of iPods sold in one week. The price function (demand function) is $p(x) = 100 - \frac{2}{30}(x - 500) = \frac{400}{3} - \frac{2}{30}x$.   (b). The revenue function is $R(x) = xp(x) = \frac{400}{3}x - \frac{2}{30}x^2$. When $x = 1000$, it has the maximum revenue income.   (c). The profit function is $P(x) = R(x) - C(x) = \frac{250}{3}x - \frac{2}{30}x^2 - 2000$. When $x = 625$, it has the maximum profit.

(a) Sketch the graph of the function;

(b) Find what value of x is this function NOT differentiable? Why?

7. (14pt) Let $f(x) = \frac{x-1}{x^2-x}$.

(a) Domain;

(b) Intercepts;

(c) Asymptotes;

(d) Critical numbers, increasing/decreasing intervals, local maximum/minimum, if any;

(e) Concavity, inflection points, if any;

(f) Absolute maximum/minimum if any, and then use the above information to sketch the graph of $f(x)$.

8. (14pts) Futureshop has been selling 500 iPods a week at $100 each. A market survey indicates that for each $20 rebate offered to the buyer, the number of sets sold will increase by 300 per week.

(a) Find the demand function;

(b) How large a rebate should Futureshop offer the buyers in order to maximize its revenue?

(c) If its weekly cost function is $C(x) = 2,000 + 50x$, how should the Futureshop set the size of the rebate in order to maximize its profit?

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