MATH 203/2 FALL 2006
ASSIGNMENT 3 SOLUTIONS

Section 2.5

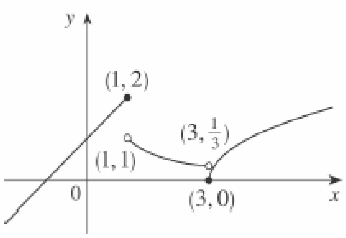
10. $\lim_{x\to 4} f(x) = \lim_{x\to 4}\left(x^2+\sqrt{7-x}\right) = \lim_{x\to 4} x^2 + \sqrt{\lim_{x\to 4} 7 - \lim_{x\to 4} x} = 4^2+\sqrt{7-4} = 16+\sqrt{3} = f(4)$.
By the definition of continuity, f is continuous at a=4.

20. $f(x) = \begin{cases} 1+x^2 & \text{if } x<1 \\ 4-x & \text{if } x\geq 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1+x^2) = 1+1^2 = 2$ and
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(4-x) = 4-1 = 3$.
Thus, f is discontinuous at 1 because $\lim_{x\to 1} f(x)$ does not exist.

38. $f(x) = \begin{cases} x+1 & \text{if } x\leq 1 \\ 1/x & \text{if } 1<x<3 \\ \sqrt{x-3} & \text{if } x\geq 3 \end{cases}$

f is continuous on $(-\infty,1)$, $(1,3)$, and $(3,\infty)$, where it is a polynomial, a rational function, and a composite of a root function with a polynomial, respectively. Now $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x+1) = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(1/x) = 1$, so f is discontinuous at 1.



Since $f(1)=2$, f is continuous from the left at 1. Also, $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(1/x) = 1/3$, and $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}\sqrt{x-3} = 0 = f(3)$, so f is discontinuous at 3 , but it is continuous from the right at 3.

42. The functions x^2-c^2 and $cx+20$, considered on the intervals $(-\infty,4)$ and $[4,\infty)$ respectively, are continuous for any value of c. So the only possible discontinuity is at $x=4$. For the function to be continuous at $x=4$, the left–hand and right–hand limits must be the same. Now $\lim_{x\to 4^-} g(x)=\lim_{x\to 4^-}\left(x^2-c^2\right)=16-c^2$ and $\lim_{x\to 4^+} g(x)=\lim_{x\to 4^+}(cx+20)=4c+20=g(4)$. Thus, $16-c^2=4c+20\Leftrightarrow c^2+4c+4=0\Leftrightarrow c=-2$.

Section 2.7

8. Using (1),

$$m=\lim_{x\to 4}\frac{\sqrt{2x+1}-\sqrt{2(4)+1}}{x-4}=\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{x-4}\cdot\frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}$$

$$=\lim_{x\to 4}\frac{(2x+1)-3^2}{(x-4)(\sqrt{2x+1}+3)}=\lim_{x\to 4}\frac{2(x-4)}{(x-4)(\sqrt{2x+1}+3)}$$

$$=\lim_{x\to 4}\frac{2}{(\sqrt{2x+1}+3)}=\frac{2}{3+3}=\frac{1}{3}\ .$$

Tangent line: $y-3=\frac{1}{3}(x-4)\Leftrightarrow y-3=\frac{1}{3}x-\frac{4}{3}\ \Leftrightarrow y=\frac{1}{3}x+\frac{5}{3}$

18. **(a)**

$$v(1)=\lim_{h\to 0}\frac{H(1+h)-H(1)}{h}$$

$$=\lim_{h\to 0}\frac{\left(58+58h-0.83-1.66h-0.83h^2\right)-57.17}{h}=\lim_{h\to 0}(56.34-0.83h)=56.34\text{ m / s}$$

(b)

$$v(a) = \lim_{h \to 0} \frac{H(a+h) - H(a)}{h}$$

$$= \lim_{h \to 0} \frac{\left(58a + 58h - 0.83a^2 - 1.66ah - 0.83h^2\right) - \left(58a - 0.83a^2\right)}{h}$$

$$= \lim_{h \to 0} (58 - 1.66a - 0.83h) = 58 - 1.66a \text{ m / s}$$

(c) The arrow strikes the moon when the height is 0 , that is, $58t - 0.83t^2 = 0 \Leftrightarrow t(58 - 0.83t) = 0 \Leftrightarrow$ $t = \frac{58}{0.83} \approx 69.9$ s (since t can't be 0).

(d) Using the time from part (c), $v\left(\frac{58}{0.83}\right) = 58 - 1.66\left(\frac{58}{0.83}\right) = -58$ m / s. Thus, the arrow will have a velocity of -58 m / s.

20. **(a)** The average velocity between times t and $t+h$ is

$$\frac{s(t+h) - s(t)}{(t+h) - t} = \frac{(t+h)^2 - 8(t+h) + 18 - \left(t^2 - 8t + 18\right)}{h}$$

$$= \frac{t^2 + 2th + h^2 - 8t - 8h + 18 - t^2 + 8t - 18}{h} = \frac{2th + h^2 - 8h}{h}$$

$$= (2t + h - 8) \text{ m/s}$$

(i) $[3,4]$: $t=3$, $h=4-3=1$, so the average velocity is $2(3)+1-8=-1$ m / s.

(ii) $[3.5,4]$: $t=3.5$, $h=0.5$, so the average velocity is $2(3.5)+0.5-8=-0.5$ m / s.

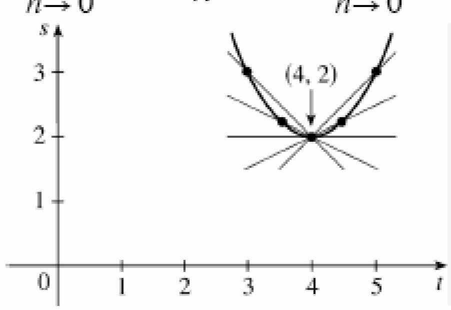
(iii) $[4,5]$: $t=4$, $h=1$, so the average velocity is $2(4)+1-8=1$ m / s.

(iv) $[4,4.5]$: $t=4$, $h=0.5$, so the average velocity is $2(4)+0.5-8=0.5$ m / s.

(b)

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h \to 0} (2t + h - 8) = 2t - 8 \text{ , so } v(4) = 0.$$

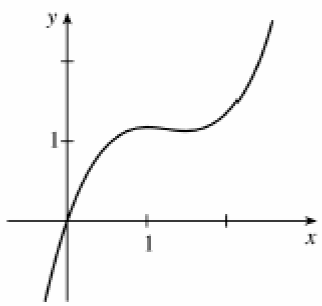
(c)



Section 2.8

2. As h decreases, the line PQ becomes steeper, so its slope increases. So $0 < \frac{f(4)-f(2)}{4-2} < \frac{f(3)-f(2)}{3-2} < \lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$. Thus, $0 < \frac{1}{2}[f(4)-f(2)] < f(3)-f(2) < f'(2)$.

6.



18.

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\sqrt{3(a+h)+1}-\sqrt{3a+1}}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{3a+3h+1}-\sqrt{3a+1})(\sqrt{3a+3h+1}+\sqrt{3a+1})}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$= \lim_{h\to 0}\frac{(3a+3h+1)-(3a+1)}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})} = \lim_{h\to 0}\frac{3h}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$= \lim_{h\to 0}\frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}} = \frac{3}{2\sqrt{3a+1}}$$

20. By Definition 2, $\lim_{h\to 0}\frac{\sqrt[4]{16+h}-2}{h} = f'(16)$, where $f(x)=\sqrt[4]{x}$ and $a=16$. *Or* : By Definition 2, $\lim_{h\to 0}\frac{\sqrt[4]{16+h}-2}{h} = f'(0)$, where $f(x)=\sqrt[4]{16+x}$ and $a=0$.

26.

$$v(2)=f^{/}(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{[2(2+h)^3-(2+h)+1]-[2(2)^3-2+1]}{h}$$

$$=\lim_{h\to 0}\frac{(2h^3+12h^2+24h+16-2-h+1)-15}{h}$$

$$=\lim_{h\to 0}\frac{2h^3+12h^2+23h}{h}=\lim_{h\to 0}(2h^2+12h+23)=23m/s$$