MATH 203/2 FALL 2006
ASSIGNMENTS 10, 11 (WEEKS 11, 12) SOLUTIONS

Section 4.4

8. $\lim\limits_{x\to 1} \dfrac{x^a-1}{x^b-1} \overset{H}{=} \lim\limits_{x\to 1} \dfrac{ax^{a-1}}{bx^{b-1}} = \dfrac{a}{b}$

18. $\lim\limits_{x\to\infty} \dfrac{\ln\ln x}{x} \overset{H}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \lim\limits_{x\to\infty} \dfrac{1}{x\ln x} = 0$

26. $\lim\limits_{x\to 0} \dfrac{\sin x - x}{x^3} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{3x^2} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-\sin x}{6x} \overset{H}{=} \lim\limits_{x\to 0} \dfrac{-\cos x}{6} = -\dfrac{1}{6}$

38. $\lim\limits_{x\to -\infty} x^2 e^x = \lim\limits_{x\to -\infty} \dfrac{x^2}{e^{-x}} \overset{H}{=} \lim\limits_{x\to -\infty} \dfrac{2x}{-e^{-x}} \overset{H}{=} \lim\limits_{x\to -\infty} \dfrac{2}{e^{-x}} = \lim\limits_{x\to -\infty} 2e^x = 0$

42. $\lim\limits_{x\to \pi/4} (1-\tan x)\sec x = (1-1)\sqrt{2} = 0$. L'Hospital's Rule does not apply.

60. $y=(\cos 3x)^{5/x} \Rightarrow \ln y = \dfrac{5}{x}\ln(\cos 3x) \Rightarrow \lim\limits_{x\to 0} \ln y = 5\lim\limits_{x\to 0} \dfrac{\ln(\cos 3x)}{x} \overset{H}{=} 5\lim\limits_{x\to 0} \dfrac{-3\tan 3x}{1} = 0$,

so $\lim\limits_{x\to 0} (\cos 3x)^{5/x} = e^0 = 1$.

Section 4.5

6. $y=f(x)=x(x+2)^3$ **A.** $D=R$ **B.** $y-$ intercept: $f(0)=0$; x –intercepts: $f(x)=0 \Leftrightarrow x=-2,0$ **C.** No symmetry **D.** No asymptote **E.** $f'(x)=3x(x+2)^2+(x+2)^3=(x+2)^2[3x+(x+2)]=(x+2)^2(4x+2)$. $f'(x)>0 \Leftrightarrow x>-\dfrac{1}{2}$, and $f'(x)<0 \Leftrightarrow x<-2$ or $-2<x<-\dfrac{1}{2}$, so f is increasing on $\left(-\dfrac{1}{2},\infty\right)$ and decreasing on $(-\infty,-2)$ and $\left(-2,-\dfrac{1}{2}\right)$ [Hence f is decreasing on $\left(-\infty,-\dfrac{1}{2}\right)$ by the analogue of Exercise 4.3.65 for decreasing functions.]

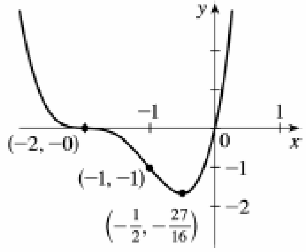
lt;b>F. Local minimum value $f\left(-\frac{1}{2}\right)=-\frac{27}{16}$, no local maximum

G.

$$\begin{aligned} f''(x) &= (x+2)^2(4)+(4x+2)(2)(x+2) \\ &= 2(x+2)\left[(x+2)(2)+4x+2\right] \\ &= 2(x+2)(6x+6)=12(x+1)(x+2) \end{aligned}$$

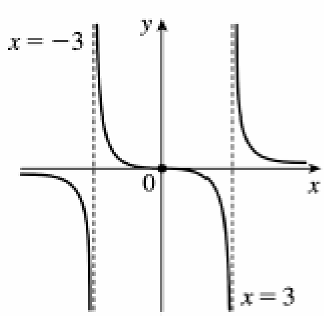
$f''(x)<0 \Leftrightarrow -2<x<-1$, so f is CD on $(-2,-1)$ and CU on $(-\infty,-2)$ and $(-1,\infty)$. IP at $(-2,0)$ and $(-1,-1)$

H.



12. $y=f(x)=x/\left(x^2-9\right)$ **A.** $D=\{x \mid x\neq\pm 3\}=(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ **B.** x –intercept $=0$, y– intercept $=f(0)=0$. **C.** $f(-x)=-f(x)$, so f is odd; the curve is symmetric about the origin. **D.** $\lim_{x\to\pm\infty}\frac{x}{x^2-9}=0$, so $y=0$ is a HA. $\lim_{x\to 3^+}\frac{x}{x^2-9}=\infty$, $\lim_{x\to 3^-}\frac{x}{x^2-9}=-\infty$, $\lim_{x\to -3^+}\frac{x}{x^2-9}=\infty$, $\lim_{x\to -3^-}\frac{x}{x^2-9}=-\infty$, so $x=3$ and $x=-3$ are VA. **E.** $f'(x)=\frac{\left(x^2-9\right)-x(2x)}{\left(x^2-9\right)^2}=-\frac{x^2+9}{\left(x^2-9\right)^2}<0$ ($x\neq\pm 3$) so f is decreasing on $(-\infty,-3)$, $(-3,3)$, and $(3,\infty)$. **F.** No extreme values **G.** $f''(x)=-\frac{2x\left(x^2-9\right)^2-\left(x^2+9\right)\cdot 2\left(x^2-9\right)(2x)}{\left(x^2-9\right)^4}=\frac{2x\left(x^2+27\right)}{\left(x^2-9\right)^3}>0$ when $-3<x<0$ or $x>3$, so f is CU on $(-3,0)$ and $(3,\infty)$; CD on $(-\infty,-3)$ and $(0,3)$ > IP at $(0,0)$

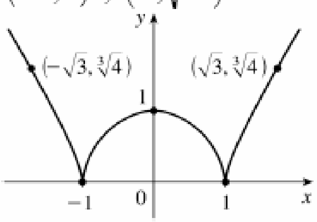
H.



30. $y=f(x)=\sqrt[3]{\left(x^2-1\right)^2}=\left(x^2-1\right)^{2/3}$ **A.** $D=R$ **B.** x –intercepts ± 1 , y –intercept 1 **C.** $f(-x)=f(x)$, so the curve is symmetric about the y– axis. **D.** $\lim_{x\to\pm\infty}\left(x^2-1\right)^{2/3}=\infty$, no asymptote **E.**

$f'(x)=\frac{4}{3}x\left(x^2-1\right)^{-1/3}\Rightarrow f'(x)>0\Leftrightarrow x>1$ or $-1<x<0$, $f'(x)<0\Leftrightarrow x<-1$ or $0<x<1$. So f is increasing on $(-1,0)$, $(1,\infty)$ and decreasing on $(-\infty,-1)$, $(0,1)$. **F.** Local minimum values $f(-1)=f(1)=0$, local maximum value $f(0)=1$ **G.** $f''(x)=\frac{4}{3}\left(x^2-1\right)^{-1/3}+\frac{4}{3}x\left(-\frac{1}{3}\right)\left(x^2-1\right)^{-4/3}(2x)$

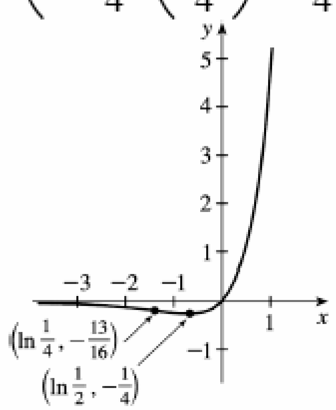
$=\frac{4}{9}\left(x^2-3\right)\left(x^2-1\right)^{-4/3}>0\Leftrightarrow|x|>\sqrt{3}$ so f is CU on $(-\infty,-\sqrt{3})$, $(\sqrt{3},\infty)$ and CD on $(-\sqrt{3},-1)$, $(-1,1)$, $(1,\sqrt{3})$. IPs at $\left(\pm\sqrt{3},\sqrt[3]{4}\right)$ **H.**



42. $y=f(x)=e^{2x}-e^x$ **A.** $D=R$ **B.** y– intercept: $f(0)=0$; x –intercepts: $f(x)=0\Rightarrow e^{2x}=e^x\Rightarrow e^x=1\Rightarrow x=0$. **C.** No symmetry **D.** $\lim_{x\to-\infty}e^{2x}-e^x=0$, so $y=0$ is a HA. No VA. **E.** $f'(x)=2e^{2x}-e^x=e^x\left(2e^x-1\right)$, so $f'(x)>0$ $\Leftrightarrow e^x>\frac{1}{2}\Leftrightarrow x>\ln\frac{1}{2}=-\ln 2$ and $f'(x)<0\Leftrightarrow e^x<\frac{1}{2}\Leftrightarrow x<\ln\frac{1}{2}$, so f is decreasing on $\left(-\infty,\ln\frac{1}{2}\right)$ and increasing on $\left(\ln\frac{1}{2},\infty\right)$. **F.** Local minimum value $f\left(\ln\frac{1}{2}\right)=e^{2\ln(1/2)}-e^{\ln(1/2)}=\left(\frac{1}{2}\right)^2-\frac{1}{2}=-\frac{1}{4}$

G. $f''(x)=4e^{2x}-e^x=e^x\left(4e^x-1\right)$, so $f''(x)>0\Leftrightarrow$
$e^x>\frac{1}{4}\Leftrightarrow x>\ln\frac{1}{4}$ and $f''(x)<0\Leftrightarrow x<\ln\frac{1}{4}$.

H. Thus, f is CD on $\left(-\infty,\ln\frac{1}{4}\right)$ and CU on $\left(\ln\frac{1}{4},\infty\right)$. f has an IP at $\left(\ln\frac{1}{4},\left(\frac{1}{4}\right)^2-\frac{1}{4}\right)=\left(\ln\frac{1}{4},-\frac{3}{16}\right)$.



Section 4.7

2. The two numbers are $x+100$ and x. Minimize $f(x)=(x+100)x=x^2+100x$. $f'(x)=2x+100=0 \Rightarrow x=-50$. Since $f''(x)=2>0$, there is an absolute minimum at $x=-50$. The two numbers are 50 and −50.

4. Let $x>0$ and let $f(x)=x+1/x$. We wish to minimize $f(x)$. Now $f'(x)=1-\frac{1}{x^2}=\frac{1}{x^2}\left(x^2-1\right)=\frac{1}{x^2}(x+1)(x-1)$, so the only critical number in $(0,\infty)$ is 1.

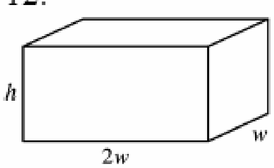
$f'(x)<0$ for $0<x<1$ and $f'(x)>0$ for $x>1$, so f has an absolute minimum at $x=1$, and $f(1)=2$.

Or: $f''(x)=2/x^3>0$ for all $x>0$, so f is concave upward everywhere and the critical point $(1,2)$ must correspond to a local minimum for f.

6. If the rectangle has dimensions x and y , then its area is $xy=1000 \text{ m}^2$, so $y=1000/x$. The perimeter $P=2x+2y=2x+2000/x$. We wish to minimize the function $P(x)=2x+2000/x$ for $x>0$.
$P'(x)=2-2000/x^2=\left(2/x^2\right)\left(x^2-1000\right)$, so the only critical number in the domain of P is $x=\sqrt{1000}$.
$P''(x)=4000/x^3>0$, so P is concave upward throughout its domain and $P\left(\sqrt{1000}\right)=4\sqrt{1000}$ is an absolute minimum value. The dimensions of the rectangle with minimal perimeter are
$x=y=\sqrt{1000}=10\sqrt{10}$ m.
(The rectangle is a square.)

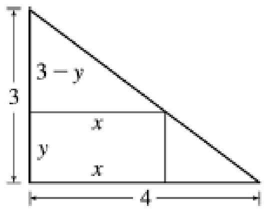
10. Let b be the length of the base of the box and h the height. The volume is $32{,}000=b^2h \Rightarrow h=32{,}000/b^2$. The surface area of the open box is
$S=b^2+4hb=b^2+4(32{,}000/b^2)b=b^2+4\,(32{,}000)\,/b$. So $S'(b)=2b-4(32{,}000)/b^2=2\left(b^3-64{,}000\right)/b^2=0 \Leftrightarrow$ $b=\sqrt[3]{64{,}000}=40$. This gives an absolute minimum since $S'(b)<0$ if $0<b<40$ and $S'(b)>0$ if $b>40$.
The box should be $40\times 40\times 20$.

12.



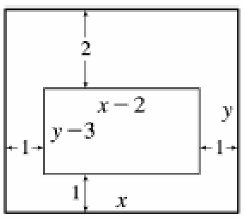
$V=lwh \Rightarrow 10=(2w)(w)h=2w^2h$, so $h=5/w^2$. The cost is $10\left(2w^2\right)+6\,[2(2wh)+2(hw)]=20w^2+36wh$, so $C(w)=20w^2+36w\left(5/w^2\right)=20w^2+180/w$. $C'(w)=40w-180/w^2=40\left(w^3-\frac{9}{2}\right)\Big/ w^2 \Rightarrow w=\sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w=\sqrt[3]{\frac{9}{2}}$ since $C'(w)<0$ for $0<w<\sqrt[3]{\frac{9}{2}}$ and $C'(w)>0$ for $w>\sqrt[3]{\frac{9}{2}}$. $C\left(\sqrt[3]{\frac{9}{2}}\right)=20\left(\sqrt[3]{\frac{9}{2}}\right)^2+\frac{180}{\sqrt[3]{9/2}}\approx \$163.54$.

24.



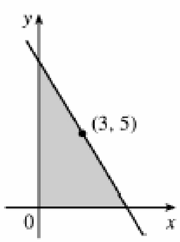
The rectangle has area xy. By similar triangles $\frac{3-y}{x}=\frac{3}{4} \Rightarrow -4y+12=3x$ or $y=-\frac{3}{4}x+3$. So the area is $A(x)=x\left(-\frac{3}{4}x+3\right)=-\frac{3}{4}x^2+3x$ where $0\le x\le 4$. Now $0=A'(x)=-\frac{3}{2}x+3 \Rightarrow x=2$ and $y=\frac{3}{2}$. Since $A(0)=A(4)=0$, the maximum area is $A(2)=2\left(\frac{3}{2}\right)=3$ cm^2.

30.



$xy=180$, so $y=180/x$. The printed area is $(x-2)(y-3)=(x-2)(180/x-3)=186-3x-360/x=A(x)$. $A'(x)=-3+360/x^2=0$ when $x^2=120 \Rightarrow x=2\sqrt{30}$. This gives an absolute maximum since $A'(x)>0$ for $0<x<2\sqrt{30}$ and $A'(x)<0$ for $x>2\sqrt{30}$. When $x=2\sqrt{30}$, $y=180/(2\sqrt{30})$, so the dimensions are $2\sqrt{30}$ in. and $90/\sqrt{30}$ in.

44.



The line with slope m (where $m<0$) through $(3,5)$ has equation $y-5=m(x-3)$ or $y=mx+(5-3m)$. The $y-$ intercept is $5-3m$ and the $x-$ intercept is $-5/m+3$. So the triangle has area $A(m)=\frac{1}{2}(5-3m)(-5/m+3)=15-25/(2m)-\frac{9}{2}m$. Now $A'(m)=\frac{25}{2m^2}-\frac{9}{2}=0 \Leftrightarrow m^2=\frac{25}{9} \Rightarrow m=-\frac{5}{3}$ (since $m<0$).

$A''(m)=-\frac{25}{m^3}>0$, so there is an absolute minimum when $m=-\frac{5}{3}$. Thus, an equation of the line is $y-5=-\frac{5}{3}(x-3)$ or $y=-\frac{5}{3}x+10$.