CHAMPLAIN COLLEGE – ST.-LAMBERT

Review Questions for Test 4

1. Which of the following numbers are natural, integer, rational, irrational?

$$-2, \quad 2.2, \quad \frac{1}{2}, \quad 0, \quad \sqrt{2}, \quad \frac{\sqrt{8}}{\sqrt{2}}, \quad \pi, \quad \sqrt{4}.$$

2. Simplify:

   (a) $\dfrac{\sqrt{8}x^{100}y^{20}}{\sqrt{2}x^{45}y^{-20}}$;

   (b) $\dfrac{3x^2+6x}{x^2-9} \div \dfrac{x^2-4}{x-3}$;

   (c) $\dfrac{1}{x^2+4x-5} - \dfrac{1}{x^2-1}$;

   (d) $\dfrac{\frac{a}{b^2} - \frac{b}{a^2}}{\frac{1}{b} - \frac{1}{a}}$.

3. Find the equations of the lines:

   (a) l_1: passes through $(1,0)$ and $(0,1)$;

   (b) l_2: passes through $(1,1)$, and is parallel to the line l_1;

   (c) l_3: passes through $(1,1)$, and is perpendicular to the line l_1.

4. Factor:

   (a) x^2-9x+8;

   (b) $3x^2+4x-4$.

5. Let $f(x) = \dfrac{1}{x^3-2x^2-3x}$. Find:

   (a) domain;

   (b) $f(1)$ and $f(2)$.

6. Solve system of equations:

$$\text{(a)} \quad \begin{cases} x-2y=-3 \\ 2x-y=0 \end{cases} \qquad\qquad \text{(b)} \quad \begin{cases} x+2y-z=4 \\ 2x-y+z=1 \\ 3x+y+z=6. \end{cases}$$

7. Rationalize the denominator:

   (a) $\dfrac{1}{2-3\sqrt{2}}$

   (b) $\dfrac{1}{a\sqrt{b}-b\sqrt{a}}$.

8. Solve equations:

   (a) $|3x+4| = |5-2x|$

   (b) $x^2 - 5x - 6 = 0$

   (c) $\dfrac{1}{4x-1} = \dfrac{2}{x+6}$

   (d) $\sqrt{2x+9} = \sqrt{2x} + 1$.

9. Solve inequalities:

   (a) $|3x+\ | < 5$

   (b) $|2x-1| > 3$.

Solutions to Review Questions for Test 4

Solution to Q.1: Notice that $\frac{\sqrt{8}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2$ and $\sqrt{4} = 2$. So, we have

natural numbers: $\frac{\sqrt{8}}{\sqrt{2}}$, $\sqrt{4}$

integers: -2, 0, $\frac{\sqrt{8}}{\sqrt{2}}$, $\sqrt{4}$

rational numbers: -2, 2.2, $\frac{1}{2}$, 0, $\frac{\sqrt{8}}{\sqrt{2}}$, $\sqrt{4}$

irrational numbers: $\sqrt{2}$, π.

Solution to Q.2(a):

$$\frac{\sqrt{8}x^{100}y^{20}}{\sqrt{2}x^{45}y^{-20}} = \frac{2\sqrt{2}}{\sqrt{2}} \cdot \frac{x^{100}}{x^{45}} \cdot \frac{y^{20}}{y^{-20}} = 2x^{100-45}y^{20-(-20)} = 2x^{55}y^{40}.$$

Solution to Q.2(b):

$$\frac{3x^2+6x}{x^2-9} \div \frac{x^2-4}{x-3} = \frac{3x(x+2)}{x^2-3^2} \cdot \frac{x-3}{x^2-2^2}$$

$$= \frac{3x(x+2)}{(x+3)(x-3)} \cdot \frac{x-3}{(x-2)(x+2)} = \frac{3x}{(x+3)(x-2)}.$$

Solution to Q.2(c):

$$\frac{1}{x^2+4x-5} - \frac{1}{x^2-1} = \frac{1}{(x+5)(x-1)} - \frac{1}{(x-1)(x+1)}$$

$$= \frac{1}{(x+5)(x-1)} \cdot \frac{(x+1)}{(x+1)} - \frac{1}{(x-1)(x+1)} \cdot \frac{(x+5)}{(x+5)}$$

$$= \frac{(x+1)}{(x+5)(x-1)(x+1)} - \frac{(x+5)}{(x+5)(x-1)(x+1)} = \frac{(x+1)-(x+5)}{(x+5)(x-1)(x+1)}$$

$$= \frac{x+1-x-5}{(x+5)(x-1)(x+1)} = -\frac{4}{(x+5)(x-1)(x+1)}.$$

Solution to Q.2(d):

$$\frac{\dfrac{a}{b^2}-\dfrac{b}{a^2}}{\dfrac{1}{b}-\dfrac{1}{a}} = \left(\frac{a}{b^2}-\frac{b}{a^2}\right) \div \left(\frac{1}{b}-\frac{1}{a}\right)$$

$$= \left(\frac{a}{b^2}\cdot\frac{a^2}{a^2}-\frac{b}{a^2}\cdot\frac{b^2}{b^2}\right) \div \left(\frac{1}{b}\cdot\frac{a}{a}-\frac{1}{a}\cdot\frac{b}{b}\right)$$

$$= \left(\frac{a^3}{a^2b^2}-\frac{b^3}{a^2b^2}\right) \div \left(\frac{a}{ab}-\frac{b}{ab}\right) = \frac{a^3-b^3}{a^2b^2} \div \frac{a-b}{ab}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{a^2b^2}\cdot\frac{ab}{a-b} = \frac{a^2+ab+b^2}{ab}.$$

Solution to Q.3(a): Its slope is:

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{1-0}{0-1} = -1.$$

By the point-slope form, we have

$$l_1: \qquad \frac{y-y_1}{x-x_1} = m_1, \quad i.e., \quad \frac{y-0}{x-1} = -1, \quad i.e., \qquad y = -x+1.$$

Solution to Q.3(b): Since $l_1 \parallel l_2$, we have

$$m_2 = m_1 = -1.$$

Then,

$$l_2: \qquad \frac{y-y_1}{x-x_1} = m_2, \quad i.e., \quad \frac{y-1}{x-1} = -1, \quad i.e., \qquad y = -x+2.$$

Solution to Q.3(c): Since $l_3 \perp l_1$, we have

$$m_1m_3 = -1, \quad m_3 = -\frac{1}{m_1} = 1.$$

Then,

$$l_3: \qquad \frac{y-y_1}{x-x_1} = m_3, \quad i.e., \quad \frac{y-1}{x-1} = 1, \quad i.e., \qquad y = x.$$

Solution to Q.4(a): By the pq-method, we have

$$x^2-9x+8 = (x-8)(x-1).$$

Solution to Q.4(b): By the ac-method, we have

$$3x^2 4x - 4 = (3x-2)(x+2).$$

Solution to Q.5(a): To find the domain, we need the denominator of the expression as non-zero,

$$x^3 - 2x^2 - 3x \neq 0, \quad i.e., \; x(x^2 - 2x - 3) \neq 0, \quad i.e., \quad x(x-3)(x+1) \neq 0$$

which gives

$$x \neq 0, \quad x \neq 3, \quad \text{and} \quad x \neq -1.$$

Thus, the domain is

$$D = (-\infty, -1) \cup (-1, 0) \cup (0, 3) \cup (3, \infty).$$

Solution to Q.5(b): To evaluate $f(1)$ and $f(2)$, we have

$$f(1) = \frac{1}{1^3 - 2(1^2) - 3(1)} = -\frac{1}{4}, \qquad f(2) = \frac{1}{2^3 - 2(2^2) - 3(2)} = -\frac{1}{6}.$$

Solution to Q.6(a): From the second equation, we have $y = 2x$. Substituting this into the first equation, we have

$$x - 2(2x) = -3, \quad i.e., x = 1.$$

Then, substituting $x = 1$ back to $y = 2x$ yields $y = 2(1) = 2$. So the solution is $x = 1$ and $y = 2$.

Solution to Q.6(b): Adding the first equation to the second equation and the third equation, respectively, we obtain

$$(x + 2y - z) + (2x - y + z) = 4 + 1, \qquad 3x + y = 5$$

and

$$(x + 2y - z) + (3x + y + z) = 4 + 6, \qquad 4x + 3y = 10.$$

From $3x + y = 5$, we have $y = 5 - 3x$, and substitute this into $4x + 3y = 10$ to obtain

$$4x + 3(5 - 3x) = 10, \quad 4x + 15 - 9x = 10, \qquad x = 1.$$

Noting $x = 1$, we then have $y = 5 - 3x = 5 - 3(1) = 2$. Substituting $x = 1$ and $y = 2$ to the second equation to have

$$2(1) - 2 + z = 1, \quad z = 1.$$

So, the solution is

$$x = 1, \quad y = 2, \quad z = 1.$$

Solution to Q.7(a):

$$\frac{1}{2 - 3\sqrt{2}} = \frac{1}{2 - 3\sqrt{2}} \cdot \frac{2 + 3\sqrt{2}}{2 + 3\sqrt{2}} = \frac{2 + 3\sqrt{2}}{2^2 - (3\sqrt{2})^2} = \frac{2 + 3\sqrt{2}}{2^2 - 3^2(\sqrt{2})^2} = \frac{2 + 3\sqrt{2}}{4 - 9(2)} = -\frac{2 + 3\sqrt{2}}{14}.$$

Solution to Q.7(b):

$$\frac{1}{a\sqrt{b}-b\sqrt{a}} = \frac{1}{a\sqrt{b}-b\sqrt{a}} \cdot \frac{a\sqrt{b}+b\sqrt{a}}{a\sqrt{b}+b\sqrt{a}}$$

$$= \frac{a\sqrt{b}+b\sqrt{a}}{(a\sqrt{b})^2-(b\sqrt{a})^2} = \frac{a\sqrt{b}+b\sqrt{a}}{a^2(\sqrt{b})^2-b^2(\sqrt{a})^2}$$

$$= \frac{a\sqrt{b}+b\sqrt{a}}{a^2b-b^2a} = \frac{a\sqrt{b}+b\sqrt{a}}{ab(a-b)}.$$

Solution to Q.8(a): The equation $|3x+4| = |5-2x|$ is equivalent to

$$3x+4 = 5-2x \qquad \text{or} \qquad 3x+4 = -(5-2x),$$

which can be solved as

$$x = \frac{1}{6} \qquad \text{or} \qquad x = -9.$$

Solution to Q.8(b):

$$(x-6)(x+1) = 0, \qquad x-6=0 \quad \text{or} \quad x+1=0, \qquad x=6 \quad \text{or} \quad x=-1.$$

Solution to Q.8(c): Multiplying the equation by $(4x-1)(x+6)$, we have

$$\frac{1}{4x-1}(4x-1)(x+6) = \frac{2}{x+6}(4x-1)(x+6),$$

namely,

$$(x+6) = 2(4x-1),$$

which can be solved as

$$x = \frac{8}{7}.$$

Check: `when` $x=\frac{8}{7}$, `LHS=`$\frac{1}{4(\frac{8}{7}-1} = \frac{7}{25}$, `and RHS=`$\frac{2}{\frac{8}{7}+6} = \frac{7}{25}$. `So,` $x=\frac{8}{7}$ `is a true solution.`

Solution to Q.8(d):

$$(\sqrt{2x+9})^2 = (\sqrt{2x}+1)^2, \qquad 2x+9 = (\sqrt{2x})^2+2\sqrt{2x}+1, \qquad 2x+9 = 2x+2\sqrt{2x}+1, \qquad 8 = 2\sqrt{2x},$$

$$\sqrt{2x} = 4, \qquad (\sqrt{2x})^2 = 4^2, \qquad 2x = 16, \qquad x = 8.$$

Check: `when` $x=8$, `LHS=`$\sqrt{2x+9} = \sqrt{2(8)+9} = \sqrt{25} = 5$, `and RHS=`$\sqrt{2x}+1 = \sqrt{2(8)}+1 = \sqrt{16}+1 = 4+1 = 5$. `So,` $x=8$ `is a true solution.`

Solution to Q.9(a): The inequality $|3x+1| < 5$ is equivalent to

$$-5 < 3x + 1 < 5, \qquad -6 < 3x < 4, \qquad -2 < x < \frac{4}{3}.$$

Solution to Q.9(b): The inequality $|3x+1| < 5$ is equivalent to

$$\text{either} \quad 2x - 1 > 3, \qquad \text{or} \qquad 2x - 1 < -3,$$

which can be solved as

$$x > 1, \qquad \text{or} \qquad x < -1.$$