CHAMPLAIN COLLEGE – ST.-LAMBERT

Review Questions for Test 3

1. Find the domain of $f(x) = \dfrac{x^2-16}{x^2-3x+2}$, and evaluate $f(0)$ and $f(3)$.

2. Simplify

   (a) $\dfrac{2x^2+20x+50}{x^2-4} \cdot \dfrac{x+2}{x+5}$ (b) $\dfrac{x}{x^2+11x+30} - \dfrac{5}{x^2+9x+20}$

   (c) $\dfrac{y^2-16}{2y+6} \div \dfrac{y-4}{y+3}$ (d) $\dfrac{1}{x+1} - \dfrac{x+2}{x^2-1} + \dfrac{3}{x-1}$

3. Simplify

   (a) $\dfrac{1-\frac{1}{x^2}}{1-\frac{1}{x}}$ (b) $\dfrac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a}-\frac{1}{b}}$.

4. Solve

   (a) $\dfrac{2}{x-1} = \dfrac{3}{x+3}$ (b) $\dfrac{7x}{x+3} + \dfrac{21}{x-3} = \dfrac{126}{x^2-9}$.

5. Simplify

   (a) $\sqrt{x^2+10x+25}$, (b) $\sqrt[4]{16x^6}$, (c) $(x^{2/3}y^{-3/4})^{12/5}$, (d) $\dfrac{\sqrt[5]{x^3y^4}}{\sqrt[5]{xy^2}}$.

6. Rationalize the denominator $\dfrac{1+\sqrt{2}}{3-5\sqrt{2}}$.

7. Solve

   (a) $\sqrt{x-6} = \sqrt{x+9} - 3$, (b) $\sqrt{x-1} + 3 = x$.

Solutions to Review Questions for Test 3

Solution to Q.1: To find the domain, we need the denominator of the expression as non-zero,

$$x^2 - 3x + 2 \neq 0, \quad i.e., \ (x-2)(x-1) \neq 0,$$

which gives

$$x \neq 1, \quad \text{and} \quad x \neq 2.$$

Thus, the domain is

$$D = (-\infty, 1) \cup (1, 2) \cup (2, \infty).$$

To evaluate $f(0)$ and $f(3)$, we have

$$f(0) = \frac{0^2 - 16}{0^2 - 3(0) + 2} = -8, \quad f(3) = \frac{3^2 - 16}{3^2 - 3(3) + 2} = -\frac{7}{2}.$$

Solution to Q.2(a):

$$\frac{2x^2 + 20x + 50}{x^2 - 4} \cdot \frac{x+2}{x+5} = \frac{2(x^2 + 10x + 25)}{x^2 - 2^2} \cdot \frac{x+2}{x+5}$$

$$= \frac{2(x+5)^2}{(x-2)(x+2)} \cdot \frac{x+2}{x+5} = \frac{2(x+5)}{x-2}.$$

Solution to Q.2(b):

$$\frac{x}{x^2 + 11x + 30} - \frac{5}{x^2 + 9x + 20} = \frac{x}{(x+5)(x+6)} - \frac{5}{(x+4)(x+5)}$$

$$= \frac{x}{(x+5)(x+6)} \cdot \frac{(x+4)}{(x+4)} - \frac{5}{(x+4)(x+5)} \cdot \frac{(x+6)}{(x+6)}$$

$$= \frac{x(x+4)}{(x+4)(x+5)(x+6)} - \frac{5(x+6)}{(x+4)(x+5)(x+6)} = \frac{x(x+4) - 5(x+6)}{(x+4)(x+5)(x+6)}$$

$$= \frac{x^2 + 4x - (5x + 30)}{(x+4)(x+5)(x+6)} = \frac{x^2 + 4x - 5x - 30)}{(x+4)(x+5)(x+6)}$$

$$= \frac{x^2 - x - 30)}{(x+4)(x+5)(x+6)} = \frac{(x+5)(x-6)}{(x+4)(x+5)(x+6)}$$

$$= \frac{x-6}{(x+4)(x+6)}.$$

Solution to Q.2(c):

$$\frac{y^2-16}{2y+6} \div \frac{y-4}{y+3} = \frac{y^2-4^2}{2(y+3)} \cdot \frac{y+3}{y-4}$$

$$= \frac{(y-4)(y+4)}{2(y+3)} \cdot \frac{y+3}{y-4} = \frac{y+4}{2}.$$

Solution to Q.2(d):

$$\frac{1}{x+1} - \frac{x+2}{x^2-1} + \frac{3}{x-1} = \frac{1}{x+1} - \frac{x+2}{(x+1)(x-1)} + \frac{3}{x-1}$$

$$= \frac{1}{x+1} \cdot \frac{(x-1)}{(x-1)} - \frac{x+2}{(x+1)(x-1)} + \frac{3}{x-1} \cdot \frac{(x+1)}{(x+1)}$$

$$= \frac{(x-1)}{(x+1)(x-1)} - \frac{x+2}{(x+1)(x-1)} + \frac{3(x+1)}{(x+1)(x-1)}$$

$$= \frac{(x-1)-(x+2)+3(x+1)}{(x-1)(x+1)} = \frac{x-1-x-2+3x+3}{(x-1)(x+1)}$$

$$= \frac{3x}{(x+1)(x-1)}.$$

Solution to Q.3(a):

$$\frac{1-\frac{1}{x^2}}{1-\frac{1}{x}} = \frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x}{x}-\frac{1}{x}} = \frac{\frac{x^2-1}{x^2}}{\frac{x-1}{x}} = \frac{x^2-1}{x^2} \div \frac{x-1}{x}$$

$$= \frac{x^2-1}{x^2} \cdot \frac{x}{x-1} = \frac{(x-1)(x+1)}{x^2} \cdot \frac{x}{x-1} = \frac{x+1}{x}.$$

Solution to Q.3(b):

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a}-\frac{1}{b}} = \left(\frac{1}{a^2}-\frac{1}{b^2}\right) \div \left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \left(\frac{1}{a^2}\cdot\frac{b^2}{b^2}-\frac{1}{b^2}\cdot\frac{a^2}{a^2}\right) \div \left(\frac{1}{a}\cdot\frac{b}{b}-\frac{1}{b}\cdot\frac{a}{a}\right)$$

$$= \left(\frac{b^2}{a^2b^2}-\frac{a^2}{a^2b^2}\right) \div \left(\frac{b}{ab}-\frac{a}{ab}\right) = \frac{b^2-a^2}{a^2b^2} \div \frac{b-a}{ab}$$

$$= \frac{b^2-a^2}{a^2b^2}\cdot\frac{ab}{b-a} = \frac{(b-a)(b+a)}{a^2b^2}\cdot\frac{ab}{b-a} = \frac{b+a}{ab}.$$

Solution to Q.4(a): Multiplying the equation by $(x-1)(x+3)$, we have

$$\frac{2}{x-1}(x-1)(x+3) = \frac{3}{x+3}(x-1)(x+3),$$

namely,

$$2(x+3) = 3(x-1),$$

which can be solved as

$$x = 9.$$

Check: `when` $x=9$, `LHS=`$\frac{2}{9-1}=\frac{1}{4}$, `and RHS=`$\frac{3}{9+3}=\frac{1}{4}$. `So,` $x=9$ `is a true solution.`

Solution to Q.4(b): The equation is equivalent to

$$\frac{7x}{x+3}+\frac{21}{x-3} = \frac{126}{(x-3)(x+3)}.$$

Multiplying the above equation by $(x-3)(x+3)$ yields

$$7x(x-3)+21(x+3) = 126, \ i.e., \ x^2 = 9,$$

which is solved as

$$x_1 = 3, \quad x_2 = -3.$$

Check: `Whenever` $x=3$ `or` $x=-3$, `the denominator of the equation always becomes 0. This is no meaning. So, both` $x=3$ `and` $x=-3$ `are false solutions. Therefore, there is no solution.`

Solution to Q.5(a):

$$\sqrt{x^2+10x+25} = \sqrt{(x+5)^2} = |x+5|.$$

Solution to Q.5(b):

$$\sqrt[4]{16x^6} = (2^4x^6)^{\frac{1}{4}} = 2^{(4)(\frac{1}{4})}x^{(6)(\frac{1}{4})} = 2^1x^{\frac{3}{2}} = 2x\sqrt{x}.$$

Solution to Q.5(c):

$$(x^{2/3}y^{-3/4})^{12/5} = x^{(\frac{2}{3})(\frac{12}{5})}y^{(-\frac{3}{4})(\frac{12}{5})} = x^{\frac{8}{5}}y^{-\frac{9}{5}}.$$

Solution to Q.5(d):

$$\frac{\sqrt[5]{x^3y^4}}{\sqrt[5]{xy^2}} = \frac{(x^3y^4)^{\frac{1}{5}}}{(xy^2)^{\frac{1}{5}}} = \frac{x^{(3)(\frac{1}{5})}y^{(4)(\frac{1}{5})}}{x^{\frac{1}{5}}y^{(2)(\frac{1}{5})}} = \frac{x^{\frac{3}{5}}y^{\frac{4}{5}}}{x^{\frac{1}{5}}y^{\frac{2}{5}}} = x^{\frac{3}{5}-\frac{1}{5}}y^{\frac{4}{5}-\frac{2}{5}} = x^{\frac{2}{5}}y^{\frac{2}{5}}.$$

Solution to Q.6:

$$\frac{1+\sqrt{2}}{3-5\sqrt{2}} = \frac{1+\sqrt{2}}{3-5\sqrt{2}} \cdot \frac{3+5\sqrt{2}}{3+5\sqrt{2}} = \frac{(1+\sqrt{2})(3+5\sqrt{2})}{3^2-(5\sqrt{2})^2} = \frac{3+8\sqrt{2}+10}{9-50} = -\frac{13+8\sqrt{2}}{41}.$$

Solution to Q.7(a): Squaring both sides of the equation gives

$$(\sqrt{x-6})^2 = (\sqrt{x+9}-3)^2,$$

i.e.,

$$x-6 = (\sqrt{x+9})^2 - 6\sqrt{x+9} + 3^2,$$

which is

$$x-6 = x+9-6\sqrt{x+9}+9.$$

We obtain

$$6\sqrt{x+9} = 24, \quad i.e., \sqrt{x+9} = 4.$$

Squaring the above equation again, we further have

$$(\sqrt{x+9})^2 = 4^2, \quad x+9 = 16, \quad x = 5.$$

Check: `For` $x = 5$, `we get LHS=`$\sqrt{x-6} = \sqrt{5-6} = \sqrt{-1}$. `No meaning. So the equation is no solution.`

Solution to Q.7(b): From the equation $\sqrt{x-1}+3 = x$, we have

$$\sqrt{x-1} = x-3.$$

Squaring the above equation yields

$$(\sqrt{x-1})^2 = (x-3)^2, \; i.e., x-1 = x^2-6x+9, \; i.e., x^2-7x+10 = 0.$$

It can be factored as

$$(x-5)(x-2)=0.$$

So, the solution is

$$x_1=5, \quad x_2=2.$$

Check: For $x_1=5$, LHS=$\sqrt{5-1}+3=\sqrt{4}+3=2+3=5$, RHS=5. So, $x_1=5$ is a true solution.

For $x_2=2$, LHS=$\sqrt{2-1}+3=\sqrt{1}+3=1+3=4$, RHS=2, namely, LHS$\neq$RHS. Thus, $x_2=2$ is a false solution.

Therefore, the equation has only one solution $x_1=5$.