CHAMPLAIN COLLEGE – ST.-LAMBERT

Review Questions for Test 2

1. Solve the following 2×2 system of equations.

(a) $\begin{cases} x + 3y = -8 \\ 4x - 3y = 23 \end{cases}$ (b) $\begin{cases} 2x + 4y = -8 \\ y = 3x - 9 \end{cases}$

(c) $\begin{cases} y - 3x = 6 \\ 6x - 2y = -12 \end{cases}$ (d) $\begin{cases} x - 3y = -1 \\ 6y - 2x = 6 \end{cases}$

2. Solve 3×3 system of equations

$$\begin{cases} x + y + z = 180 \\ x - z = -70 \\ 2y - z = 0 \end{cases}$$

3. The perimeter of a standard tennis court used for playing doubles is 288 ft. The width of the court is 42 ft less than the length. Find the length and the width.

4. Factor

(a) $x^2 - 3x - 18$ (b) $x^8 - y^8$

(c) $x^2 - 6x + 9$ (d) $2x^2 - 5x + 2$

5. Solve

(a) $x^2 - 5x - 14 = 0,$ (b) $2x^2 + 21 = -17x.$

Solutions to Review Questions for Test 2

Solution to Q.1 (a): Multiplying the first equation by -4 and adding it to the second equation, we have

$$-15y = 55,$$

which gives

$$y = -\frac{11}{3}.$$

Then, substituting $y = -\frac{11}{3}$ to the first equation, we further obtain

$$x = -8 - 3y = -8 - (-\frac{11}{3}) = 3.$$

So the solution is $x = 3$ and $y = -\frac{11}{3}$.

Solution to Q.1 (b): Substituting the second equation $y = 3x - 9$ to the first equation, we have

$$2x + 4(3x - 9) = -6,$$

which can be solved as

$$x = 2.$$

Substituting $x = 2$ to the second equation, we then obtain

$$y = 3(2) - 9 = -3.$$

So, the solution is $x = 2$ and $y = -3$.

Solution to Q.1 (c): Dividing the second equation by -2 yields

$$y - 3x = 6,$$

which is identical to the first equation. So the system has infinitely many solutions, namely,

$$\begin{cases} x = t \\ y = 3t + 6. \end{cases} \quad (t: \text{ any number})$$

Solution to Q.1 (d): Dividing the second equation by -2 yields

$$x - 3y = -3.$$

Subtracting this new equation $x - 3y = -3$ from the first equation $x - 3y = -1$ gives

$$0 = 2.$$

This is a contradiction. So the system is no solution.

Solution to Q.2: Subtracting the second equation $x - z = -70$ from the first equation $x + y + z = 180$, we have a new equation

$$y + 2z = 250.$$

From the third equation $2y - z = 0$, we obtain

$$z = 2y.$$

Substituting $z = 2y$ to the above new equation $y + 2z = 250$, we then get

$$5y = 250, \quad i.e., \; y = 50.$$

Thus, from the third equation and the second equation, we have

$$z = 2y = 2(50) = 100, \quad \text{and} \quad x = z - 70 = 100 - 70 = 30.$$

So, the solution for this 3×3 system is:

$$x = 30, \quad y = 50, \quad z = 100.$$

Solution to Q.3: Let x and y be the width and the length of the tennis court in feet, respectively. Since the perimeter of the court is 288 ft, we have

$$2x + 2y = 288.$$

On the other hand, the width is 42 ft less than the length, so we have

$$x = y - 42.$$

Thus, we build up a 2×2 system of equations

$$\begin{cases} 2x + 2y = 288 \\ x = y - 42. \end{cases}$$

Dividing the first equation by 2, we get a new equation

$$x + y = 144.$$

Then substituting the second equation $x = y - 42$ to this new equation $x + y = 144$, we have

$$y - 42 + y = 144,$$

which gives $y = 93$. Furthermore, substituting $y = 93$ to the second equation, we finally have

$$x = y - 42 = 93 - 42 = 51.$$

So, the width of this standard tennis court is 51 ft, and its length is 93 ft.

Solution to Q.4(a):

$$\begin{aligned} x^2 - 3x - 18 &= x^2 - (6-3)x - 18 \\ &= x^2 - 6x + 3x - 18 \\ &= (x^2 - 6x) + (3x - 18) \\ &= x(x-6) + 3(x-6) \\ &= (x-6)(x+3). \end{aligned}$$

Solution to Q.4(b):

$$\begin{aligned} x^8 - y^8 &= (x^4)^2 - (y^4)^2 \\ &= (x^4 - y^4)(x^4 + y^4) \\ &= [(x^2)^2 - (y^2)^2](x^4 + y^4) \\ &= (x^2 - y^2)(x^2 + y^2)(x^4 + y^4) \\ &= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4). \end{aligned}$$

Solution to Q.4(c):

$$x^2 - 6x + 9 = x^2 - 2(x)(3) + (3)^2 = (x-3)^2.$$

Solution to Q.4(d):

$$\begin{aligned} 2x^2 - 5x + 2 &= 2x^2 - (1+4)x + 2 \\ &= 2x^2 - x - 4x + 2 \\ &= (2x^2 - x) + (-4x + 2) \\ &= x(2x-1) - 2(2x-1) \\ &= (x-2)(2x-1). \end{aligned}$$

Solution to Q.5(a):

$$x^2 - 5x - 14 = 0, \quad i.e., \quad (x-7)(x+2) = 0.$$

So,

$$x - 7 = 0, \quad \text{or} \quad x + 2 = 0,$$

which gives

$$x_1 = 7, \quad \text{or} \quad x_2 = -2.$$

Solution to Q.5(b):

$$2x^2 + 21 = -17x, \quad i.e., \quad 2x^2 + 17x + 21 = 0,$$

which is equivalent to

$$(2x+3)(x+7) = 0.$$

This can be solved as

$$2x + 3 = 0, \quad \text{or} \quad x + 7 = 0,$$

namely, $x_1 = -\frac{3}{2}$, $x_2 = -7$.