Assignment # 1

Solutions

1.

The augmented matrix of the system is:

$$\begin{pmatrix} 0 & 1 & 4 & 1 & 5 \\ 3 & -7 & 3 & -1 & -4 \\ 1 & -2 & -1 & 0 & -3 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_1}$$

$$\begin{pmatrix} 1 & -2 & -1 & 0 & -3 \\ 3 & -7 & 3 & -1 & -4 \\ 0 & 1 & 4 & +1 & 5 \end{pmatrix} \xrightarrow{R_2 - 3R_1}$$

$$\begin{pmatrix} 1 & -2 & -1 & 0 & -3 \\ 0 & -1 & 6 & -1 & 5 \\ 0 & 1 & 4 & +1 & 5 \end{pmatrix} \xrightarrow[\substack{R_3 + R_2 \\ R_1 - 2R_2}]{-R_2}$$

$$\begin{pmatrix} 1 & 0 & -13 & 2 & -13 \\ 0 & 1 & -6 & 1 & -5 \\ 0 & 0 & 10 & 0 & 10 \end{pmatrix} \xrightarrow[\substack{R_1 + \frac{13}{10} R_3 \\ R_2 + \frac{6}{10} R_3}]{\frac{1}{10} R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

therefore

$$\begin{cases} x = -2w \\ y = 1 - w \\ z = 1 \end{cases} \quad \text{ie} \quad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + w \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$w \in \mathbb{R}$

$\boxed{2}$ The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & k^2 & k+4 \end{bmatrix} \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1}$$

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & -1 & -2 \\ 0 & -2 & k^2-3 & k-5 \end{pmatrix} \xrightarrow[\substack{R_1 + R_2 \\ R_3 - 2R_2}]{-R_2}$$

$$\begin{pmatrix} 1 & 0 & 0 & +1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & k^2-1 & k-1 \end{pmatrix}$$

(a) If $k^2 - 1 = 0$ and $k - 1 \neq 0$ ie $\boxed{k = -1}$

then the system is inconsistent.

(b) If $k^2 \neq 1$ ie $K \neq 1$ and $K \neq -1$ then

the system has a unique solution

(c) If $k = 1$, then the system is

equivalent to

$$\begin{cases} x = +1 \\ y = 2 - z \end{cases} \quad \text{ie} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} +1 \\ 2 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$\boxed{3}$

(a)

$$A = \begin{pmatrix} \lambda - 2 & 2 \\ 1 & \lambda - 1 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & \lambda - 1 \\ \lambda - 2 & 2 \end{pmatrix} \xrightarrow{R_2 - (\lambda-2) R_1} \begin{pmatrix} 1 & \lambda - 1 \\ 0 & 2 - (\lambda-2)(\lambda-1) \end{pmatrix}$$

If $\quad 2 - (\lambda-2)(\lambda-1) \neq 0 \quad$ ie $\quad \lambda(\lambda - 3) \neq 0 \quad$ ie

$\boxed{\lambda \neq 0}$ and $\boxed{\lambda \neq 3}$ then the homogeneous system

$\underset{\sim}{A}\, \underset{\sim}{x} = \underset{\sim}{0}$ has only the zero solution.

(b)

① If $\lambda = 0$ The system is equivalent to

$\{\, x - y = 0 \quad$ ie $\quad \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad t \in \mathbb{R}$

② If $\lambda = 3$ the system is equivalent to

$x + 2y = 0 \quad$ ie

$$\begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad t \in \mathbb{R}$$

(4)

X = Number of 1\$ bills
Y = Number of 5\$ bills
Z = Number of 10\$ bills

we have

$$\begin{cases} x + y + z = 32 \\ x + 5y + 10z = 100 \end{cases}$$

or equivalently

$$\begin{cases} x + y + z = 32 & (1) \\ 4y + 9z = 68 & (2) \end{cases}$$

Note that z is integer valued and $0 \leq z \leq 10$

and y is also integer valued,

Plug in successively $z = 0, 1, 2, 3 \ldots$ into (2)

and try to solve for y (Remember y is an integer)

The only solution is

$\underline{z = 4}$ $\underline{y = 8}$ and $x \underline{= 20}$