MATHEMATICS 133 ASSIGNMENT 2

Due in class on October 26 (for Loveys and Anderson's sections) and on October 25 (for Clay, Shahabi and Kelome's sections).

Instructions: Show all work and justify answers (even where not explicitly requested). Marks may be deducted for lack of neatness (print if necessary). The assignment mark may be based on a randomly selected problem or problems instead of the whole assignment. Therefore be sure to solve each problem. **In BLOCK CAPITALS, write your INSTRUCTOR'S LAST NAME, your LAST NAME , and your ID number, in the top right corner**.

1. Prove or disprove the following statements;
   (a) if A is invertible then $adj(A)$ is invertible
   (b) if A is invertible then A^2 is invertible
   (c) if A has a zero entry on the diagonal then A is not invertible.
   (d) if A is not invertible then for every matrix B, AB is not invertible.
   (e) if A is a nonzero 2×2 matrix such that $A^2 + A = 0$ then A is invertible.
2. Let A be a 3×3 matrix such that the sum of the entries of each row is equal to zero. Show that A is not invertible. (*Hint*: construct a 3×3 invertible matrix B such that $\det(AB) = 0$).
3. Let $\mathbf{r_1}$, $\mathbf{r_2}$ and $\mathbf{r_3}$ be the rows of a 3×3 matrix $\mathbf{A} = \begin{bmatrix} \mathbf{r_1} \\ \mathbf{r_2} \\ \mathbf{r_3} \end{bmatrix}$. Let $\mathbf{B} = \begin{bmatrix} 2\mathbf{r_1} + 3\mathbf{r_2} \\ \mathbf{r_3} + 2\mathbf{r_1} \\ \mathbf{r_2} - \mathbf{r_1} - 3\mathbf{r_3} \end{bmatrix}$
   If $\det(A) = 2$, find $\det(B)$.
4. Consider the matrix
   $$A = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{pmatrix}$$
   (a) Show that $\lambda = 1$ is an eigenvalue of A.
   (b) Is A diagonalizable?
5. Prove or disprove
   (a) If λ is an eigenvalue of A, then λ is also an eigenvalue of A^T.
   (b) If $A^2 + A = 0$, then $\lambda = 1$ cannot be an eigenvalue of A.
   (c) A 3×3 matrix cannot have four distinct eigenvalues.
   (d) An invertible matrix A can have $\lambda = 0$ as eigenvalue.
   (e) if λ is an eigenvalue of an invertible matrix A, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.