Math 240: Discrete Structures I

Practice Midterm

Instructions. The exam is 50 minutes long and contains 2 questions. You may quote any result/theorem seen in the lectures or in the assignments without proving it (unless, of course, it is what the question asks you to prove).

1. Logic.

(a) What is the negation of

\[ \forall n \in \mathbb{N} \ (n \text{ is prime } \Rightarrow n^2 + 1 \text{ is even}) \]

(b) Use a truth table to decide whether or not the following implication is a tautology:

\[ (p \land q) \Rightarrow (p \Rightarrow q) \]

(c) Give a proof to show that the following implication is a tautology:

\[ \overline{(p \Rightarrow q)} \Rightarrow p \]

2. Number Theory.

(a) Give the prime factorisation of 312.

(b) Use Euclid’s algorithm to find \( \text{gcd}(312, 97) \).

(c) Find integers \( s \) and \( t \) such that \( \text{gcd}(312, 97) = 312s + 97t \).

(d) Solve the modular equation

\[ 312x \equiv 3 \mod 97 \]