MATH 556 - ASSIGNMENT 2

To be handed in not later than 5pm, 11th October 2007.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1. Suppose that X_1 and X_2 are continuous random variables, with joint pdf specified as

$$f_{X_1,X_2}(x_1,x_2) = f_{X_1}(x_1) f_{X_2|X_1}(x_2|x_1)$$

where, for $(x_1,x_2) \in \mathbb{R}^2$,

$$\begin{aligned} f_{X_1}(x_1) &= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x_1^2\right\} \\ f_{X_2|X_1}(x_2|x_1) &= \frac{1}{\sqrt{8\pi}} \exp\left\{-\frac{1}{8}(x_2-x_1)^2\right\}. \end{aligned}$$

(i) Show that the joint distribution of (X_1, X_2) is Multivariate Normal with parameters to be identified.

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(ii) Find the marginal distribution of X_2.

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2. Suppose that T_1 and T_2 are independent $Uniform(-1,1)$ random variables with pdfs that are constant on the interval $(-1,1)$. Let

$$R = \sqrt{T_1^2 + T_2^2}.$$

Show that, *conditional on* $R \leq 1$, the random variables

$$X = \frac{T_1}{R}\sqrt{-2\log R^2} \qquad Y = \frac{T_2}{R}\sqrt{-2\log R^2}$$

are independent, and identify their marginal distributions.

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3. The *skewness* and *kurtosis* of a probability distribution is defined in terms of the first four *central moments* of the distribution: specifically,

$$\text{Skewness} \quad : \quad \text{skw}_{f_X}[X] = \frac{m_3'}{\{m_2'\}^{3/2}}$$

$$\text{Kurtosis} \quad : \quad \text{kur}_{f_X}[X] = \frac{m_4'}{\{m_2'\}^{2}}$$

where

$$m_r' = E_{f_X}[(X-m_1)^r]$$

is the rth central moment, and $m_r = E_{f_X}[X^r]$ is the rth moment, for $r = 1,2,3,\ldots$.

Find the skewness and kurtosis if $X \sim Poisson(\lambda)$.

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