### MATH 204 - SOLUTIONS 4

For these questions we will use the method of ANOVA-F testing for nested models, and the test statistic

$$F = \frac{(SSE_R - SSE_C)/(k - g)}{SSE_C/(n - k - 1)}$$

where  $SSE_R$  is the error sum of squares for the **Reduced Model**, specified using g + 1 parameters including the intercept, and  $SSE_C$  is the error sum of squares for the **Complete Model**, specified using k + 1 parameters including the intercept.

If the reduced model is an adequate simplification of the complete model, then

$$F \sim \text{Fisher-F}(k-g, n-k-1)$$

Note here that

$$k-g = (n-g-1) - (n-k-1) = EDF_R - EDF_C$$

so the k - g quantities can be deduced directly from the EDFs.

1. For this problem, we use ANOVA and linear regression techniques, specifically multiple regression. Note that **Model** and **Vendor** are factor predictors, so we use the General Linear Model pulldown menu in SPSS.

The SPSS output for a series of models is attached; we fit in turn each of the single predictor models, then the multiple regression model with all variables included, then different models with variables and interactions included. We use inspection of p-values in ANOVA tables and  $R^2$  statistics to assess the most suitable model fit. For the analysis, price is in thousands of pounds.

Note that this is only an informal model comparison procedure; we do not use the formal ANOVA-F test comparison models developed later.

Our conclusions are summarized as follows:

- In the main effects only models (Models 1 4), **Model**, **Age**, and **Mileage** are important predictors, as all have significant *p*-values in the one-way ANOVA. Of these variables, **Model** seems to be the most important predictor, with an  $R^2$  value of 0.77. The variable **Vendor** is not significant at the  $\alpha = 0.05$  significance level (p = 0.089).
- In the multiple regression model with interaction between the two factor predictors (Model 5), **Age** and **Model** appear to be significant predictors (precise interpretation may be difficult in this unbalanced design). The  $R^2$  value is now 0.947, indicating good explanatory power.
- After checking a selection of models (Model 6 10) it seems that the best model in terms of simplicity and good explanatory power is the model

### Age + Model

No other terms appear to be significant, and also  $R^2 = 0.906$  with Adjusted  $R^2 = 0.896$ , so the explanatory power is good.

- Inspection of the residuals indicates that overall the model assumptions are met, as we see no pattern in the residuals. There may be evidence of a single outlier (the car with the highest observed price)
- Inspection of the parameter estimates indicates that price **decreases** with increasing **Age** (estimated coefficient is -1.079, standard error 0.138), and that the 500 series (**Model**=0) has the highest price, with coefficient 13.486+11.966 = 25.452.

	Model	k	EDF	SSE
M0	Null	0	19	358.670
M1	trt	4	15	160.263
M2	initialwt	1	18	16.312
M3	trt + initialwt	5	14	4.222
M4	trt + initialwt + trt . initialwt	9	10	2.834

- 2. **Oysters Data Set** Here we can fit the five models listed below. For these data n = 20. Inspection of the ANOVA tables implies that, rather than go through forward or backward selection, we might start with the model M3, trt + initialwt, and try to include or omit terms.
  - M3 vs M4

$$F = \frac{(4.222 - 2.834)/(9 - 5)}{2.834/10} = 1.224$$

From tables Fisher- $F_{0.05}(4, 10) = 3.48 > 1.224$ , so we **do not reject** M3 as an adequate simplification of M4.

• M1 vs M3

$$F = \frac{(160.263 - 4.222)/(5 - 4)}{4.222/14} = 517.462$$

From tables Fisher- $F_{0.05}(1, 14) = 4.60 < 517.462$ , so we reject M1 as an adequate simplification of M3.

• M2 vs M3

$$F = \frac{(16.312 - 4.222)/(5 - 1)}{4.222/14} = 10.022$$

From tables Fisher- $F_{0.05}(4, 14) = 3.11 < 10.022$ , so we reject M2 as an adequate simplification of M3.

Hence the most suitable model is M3. The  $R^2$  and Adjusted  $R^2$  values are 0.988 and 0.984, indicating that the fit is very good.

3. **Oranges Data Set** Here we have two regression models for Q1 and Q2 in terms of the predictors. For these data n = 36.

First for Q1 ; note that we cannot fit the interaction day . store as we do not have sufficient data, as we only have one replicate per day  $\times$  store combination.

The models are listed in the order they are presented in the output:

	Model	k	EDF	SSE
M0	Null	0	35	1622.676
M1	day + store + P1 + day . P1 + store . P1	21	14	286.786
M2	day + store + P1 + day . P1	16	19	368.863
M3	day + store + P1	11	24	447.850
M4	day + P1	6	29	686.545
M5	day + P1 + day . P1	11	24	522.153
M6	P1	1	34	1117.084

We proceed with the relevant comparisons:

• M2 vs M1

$$F = \frac{(368.863 - 286.786)/(21 - 16)}{286.786/14} = 0.801$$

From tables Fisher- $F_{0.05}(5, 14) = 2.96 > 0.801$ , so we **do not reject** M2 as an adequate simplification of M1.

• M3 vs M2

$$F = \frac{(447.850 - 368.863)/(16 - 11)}{368.863/19} = 0.814$$

From tables Fisher- $F_{0.05}(5, 19) = 2.740 > 0.814$ , so we **do not reject** M3 as an adequate simplification of M2.

• M4 vs M3

$$F = \frac{(686.545 - 447.850)/(11 - 6)}{447.850/24} = 2.558$$

From tables Fisher- $F_{0.05}(5, 24) = 2.62 > 2.558$ , so we **do not reject** M4 as an adequate simplification of M3, although the result is almost significant.

• M4 vs M5

$$F = \frac{(686.545 - 522.153)/(11 - 6)}{522.153/24} = 1.511$$

From tables Fisher- $F_{0.05}(5, 24) = 2.62 > 1.511$ , so we **do not reject** M4 as an adequate simplification of M5.

• M6 vs M4

$$F = \frac{(1117.084 - 686.545)/(6-1)}{686.54/29} = 3.63$$

From tables Fisher- $F_{0.05}(5, 29) = 2.55 < 3.63$ , so we reject M6 as an adequate simplification of M4.

Hence it seems that the model day + P1 is the most appropriate model. For this model, the  $R^2$  and Adjusted  $R^2$  values are 0.580 and 0.493 respectively, so the explanatory power of the model is only moderate.

Secondly for Q2. The models are listed in the order they are presented in the output:

	Model	k	EDF	SSE
M0	Null	0	35	2750.208
M1	day + store + P2 + day . P2 + store . P2	21	14	275.701
M2	day + store + P2 + day . P2	16	19	464.396
M3	day + store + P2	11	24	790.864
M4	day + P2	6	29	1100.743
M5	day + P2 + day . P2	11	24	845.661
M6	P2	1	34	1864.648

We proceed with the relevant comparisons

• M2 vs M1

$$F = \frac{(464.396 - 275.701)/(21 - 16)}{275.701/14} = 1.916$$

From tables Fisher- $F_{0.05}(5, 14) = 2.96 > 1.916$ , so we **do not reject** M2 as an adequate simplification of M1.

• M3 vs M2

$$F = \frac{(790.864 - 464.396)/(16 - 11)}{464.396/19} = 2.671$$

From tables Fisher- $F_{0.05}(5, 19) = 2.740 > 2.671$ , so we **do not reject** M3 as an adequate simplification of M2, although the result is almost significant.

• M4 vs M3

$$F = \frac{(1100.743 - 790.864)/(11 - 6)}{790.864/24} = 1.881$$

From tables Fisher- $F_{0.05}(5, 24) = 2.62 > 1.881$ , so we **do not reject** M4 as an adequate simplification of M3.

• M4 vs M5

$$F = \frac{(1100.743 - 845.661)/(11 - 6)}{845.661/24} = 1.448$$

From tables Fisher- $F_{0.05}(5, 24) = 2.62 > 1.448$ , so we **do not reject** M4 as an adequate simplification of M5.

• M6 vs M4

$$F = \frac{(1864.648 - 1100.743)/(6 - 1)}{1100.743/29} = 4.025$$

From tables Fisher- $F_{0.05}(5, 29) = 2.55 < 4.025$ , so we **reject** M6 as an adequate simplification of M4.

Hence it seems that the model day + P2 is the most appropriate model. For this model, the  $R^2$  and Adjusted  $R^2$  values are 0.600 and 0.517 respectively, so the explanatory power of the model is only moderate.

Note that for the two models, the estimates of the random error variance  $\sigma^2$  are given by the quantity SSE/EDF, so

Q1 : 
$$\hat{\sigma}^2 = \frac{686.545}{29} = 23.674$$
 Q2 :  $\hat{\sigma}^2 = \frac{1100.743}{29} = 37.957$ 

that is, the random error variances seem very different in the two data sets. Hence a combined analysis is **not carried out**, as this would require a common  $\sigma^2$ .

4. **Cotton Data Set** For these data n = 49. Inspection of the SPSS output suggests that we may simplify the full model

variety \* spacing \* bollwt

by dropping some of the higher order interactions. The models compared are listed below:

	Model	k	EDF	SSE
M0	Null	0	48	33.091
M1	$variety \star spacing \star bollwt$	7	41	1.730
M2	<pre>variety + spacing + bollwt + variety.bollwt</pre>	4	44	1.809
M3	variety + spacing + bollwt	3	45	2.291
M4	variety + bollwt + variety.bollwt	3	45	2.184

• M2 vs M1

$$F = \frac{(1.809 - 1.730)/(7 - 4)}{1.730/41} = 0.624$$

From tables Fisher- $F_{0.05}(3, 41) \simeq 2.84 > 0.624$ , so we **do not reject** M2 as an adequate simplification of M1.

• M3 vs M2

$$F = \frac{(2.291 - 1.809)/(4 - 3)}{1.809/44} = 11.724$$

From tables Fisher- $F_{0.05}(1, 44)$  < Fisher- $F_{0.05}(1, 40) = 4.08 < 11.724$ , so we reject M3 as an adequate simplification of M2.

• M4 vs M2

$$F = \frac{(2.184 - 1.809)/(4 - 2)}{1.809/44} = 4.561$$

From tables Fisher- $F_{0.05}(2, 44) < Fisher-F_{0.05}(2, 40) = 3.23$ , so we reject M4 as an adequate simplification of M2.

Hence the selected model is M2

variety + spacing + bollwt + variety.bollwt

Parameter estimates confirm that the response is an **increasing** function of bollwt (coefficient 0.240, standard error 0.025). The  $R^2$  and adjusted  $R^2$  of the model are large (0.945 and 0.940), so the predictive power is high. The residual plot indicates that the final model is adequate.

5. **Doses Data Set** For these data n = 24, and we have a balanced complete factorial design. We cannot fit a three-way interaction model as we do not have sufficient replicates (one observation in each combination of the  $4 \times 2 \times 3 = 24$  factor levels). Inspection of the SPSS output suggests that we may simplify the most complex model

by dropping some of the interaction terms. The models compared are listed below:

	Model	k	EDF	SSE
M0	Null	0	24	1068.958
M1	<pre>bloc+ type+ dose+ bloc. type+ bloc. dose+ type. dose</pre>	17	6	74.917
M2	bloc+ type+ dose+ bloc. dose+ type. dose	14	9	122.375
M3	bloc+ type+ dose+ type. dose	8	15	252.458
M4	bloc+ type+ dose	6	17	396.542

• M2 vs M1

$$F = \frac{(122.375 - 74.917)/(9 - 6)}{74.917/6} = 1.267$$

From tables Fisher- $F_{0.05}(3, 6) = 4.76 > 1.267$ , so we **do not reject** M2 as an adequate simplification of M1.

• M3 vs M2

$$F = \frac{(252.458 - 122.375)/(15 - 9)}{122.375/9} = 1.594$$

From tables Fisher- $F_{0.05}(6,9) = 3.37 > 1.594$ , so we **do not reject** M3 as an adequate simplification of M2.

• M4 vs M3

$$F = \frac{(396.542 - 252.458)/(8 - 6)}{252.458/15} = 4.280$$

From tables Fisher- $F_{0.05}(2, 15) = 3.68 < 4.28$ , so we reject M4 as an adequate simplification of M3.

No further terms can be dropped from the model, so we select M3

bloc + type + dose + type.dose

as the most suitable model. The  $R^2$  and adjusted  $R^2$  of the model are large (0.764 and 0.638), so the predictive power is good. The residual plot indicates that the final model is adequate.

# **SPSS Output for Exercises 4**

# Model 1: Mod

Dependent Variable: Price (1000 GBP)									
Source	Type III Sum of Squares	df	Mean Square	F	Sig.				
Corrected Model	1105.468(a)	4	276.367	45.279	.000				
Intercept	11607.038	1	11607.038	1901.661	.000				
Mod	1105.468	4	276.367	45.279	.000				
Error	299.078	49	6.104						
Total	13658.417	54							
Corrected Total	1404.546	53							

a R Squared = .787 (Adjusted R Squared = .770)

#### Dependent Variable: Price (1000 GBP)

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	9.236	.618	14.953	.000	7.994	10.477
[Mod=0]	12.843	1.070	12.005	.000	10.693	14.993
[Mod=1]	5.610	1.266	4.432	.000	3.067	8.154
[Mod=2]	9.922	.996	9.963	.000	7.921	11.923
[Mod=3]	5.648	.888	6.361	.000	3.863	7.432
[Mod=4]	0(a)				•	

a This parameter is set to zero because it is redundant.

# Model 2: Age

Dependent Variable: Price (1000 GBP)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	258.133(a)	1	258.133	11.709	.001
Intercept	4109.494	1	4109.494	186.402	.000
Age	258.133	1	258.133	11.709	.001
Error	1146.413	52	22.046		
Total	13658.417	54			
Corrected Total	1404.546	53			

a R Squared = .184 (Adjusted R Squared = .168)

### Dependent Variable: Price (1000 GBP)

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	19.409	1.422	13.653	.000	16.557	22.262
Age	-1.128	.330	-3.422	.001	-1.790	467

# Model 3: Mile

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	326.165(a)	1	326.165	15.728	.000
Intercept	5063.081	1	5063.081	244.144	.000
Mile	326.165	1	326.165	15.728	.000
Error	1078.381	52	20.738		
Total	13658.417	54			
Corrected Total	1404.546	53			

Dependent Variable: Price (1000 GBP)

a R Squared = .232 (Adjusted R Squared = .217)

Dependent Variable: Price (1000 GBP)

					95% Confidence Interval		
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound	
Intercept	19.302	1.235	15.625	.000	16.823	21.781	
Mile	209	.053	-3.966	.000	315	103	

### Model 4: Vend

Dependent Variable: Price (1000 GBP)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	209.561(a)	4	52.390	2.148	.089
Intercept	12329.637	1	12329.637	505.573	.000
Vend	209.561	4	52.390	2.148	.089
Error	1194.985	49	24.387		
Total	13658.417	54			
Corrected Total	1404.546	53			

a R Squared = .149 (Adjusted R Squared = .080)

#### Dependent Variable: Price (1000 GBP)

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	13.503	1.370	9.859	.000	10.751	16.256
[Vend=0]	3.015	2.023	1.490	.143	-1.050	7.081
[Vend=1]	5.054	2.219	2.278	.027	.595	9.514
[Vend=2]	1.925	2.141	.899	.373	-2.378	6.229
[Vend=3]	511	1.937	264	.793	-4.403	3.382
[Vend=4]	0(a)				-	

a This parameter is set to zero because it is redundant.

## Model 5: Age + Mile + Mod + Vend + Mod.Vend

Dependent Variable: Price (1000 GBP)							
Source	Type III Sum of Squares	df	Mean Square	F	Sig.		
Corrected Model	1329.511(a)	24	55.396	21.410	.000		
Intercept	1907.237	1	1907.237	737.122	.000		
Age	47.504	1	47.504	18.360	.000		
Mile	1.769	1	1.769	.684	.415		
Mod	604.015	4	151.004	58.361	.000		
Vend	14.839	4	3.710	1.434	.248		
Mod * Vend	36.082	14	2.577	.996	.482		
Error	75.035	29	2.587				
Total	13658.417	54					
Corrected Total	1404.546	53					

a R Squared = .947 (Adjusted R Squared = .902)

# Model 6: Age + Mile + Mod + Vend

Dependent Variable: Price (1000 GBP)								
Source	Type III Sum of Squares	df	Mean Square	F	Sig.			
Corrected Model	1293.428(a)	10	129.343	50.053	.000			
Intercept	2413.866	1	2413.866	934.113	.000			
Mod	888.417	4	222.104	85.949	.000			
Vend	16.608	4	4.152	1.607	.190			
Age	60.368	1	60.368	23.361	.000			
Mile	2.461	1	2.461	.952	.335			
Error	111.117	43	2.584					
Total	13658.417	54						
Corrected Total	1404.546	53						
Corrected Model Intercept Mod Vend Age Mile Error Total Corrected Total	1293.428(a) 2413.866 888.417 16.608 60.368 2.461 111.117 13658.417 1404.546	10 1 4 1 1 43 54 53	129.343 2413.866 222.104 4.152 60.368 2.461 2.584	50.053 934.113 85.949 1.607 23.361 .952	.000 .000 .190 .000 .335			

a R Squared = .921 (Adjusted R Squared = .902)

# Model 7: Age + Mod + Vend

### Dependent Variable: Price (1000 GBP)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1290.967(a)	9	143.441	55.569	.000
Intercept	2474.277	1	2474.277	958.528	.000
Mod	927.675	4	231.919	89.845	.000
Vend	18.131	4	4.533	1.756	.155
Age	123.195	1	123.195	47.726	.000
Error	113.579	44	2.581		
Total	13658.417	54			
Corrected Total	1404.546	53			

a R Squared = .919 (Adjusted R Squared = .903)

## Model 8: Age + Mod

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1272.836(a)	5	254.567	92.774	.000
Intercept	2949.842	1	2949.842	1075.032	.000
Mod	1014.703	4	253.676	92.449	.000
Age	167.368	1	167.368	60.995	.000
Error	131.710	48	2.744		
Total	13658.417	54			
Corrected Total	1404.546	53			

Dependent Variable: Price (1000 GBP)

a R Squared = .906 (Adjusted R Squared = .896)

## Model 9: Age + Mile + Mod

	,	. /			
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1276.820(a)	6	212.803	78.307	.000
Intercept	2953.826	1	2953.826	1086.941	.000
Mod	920.691	4	230.173	84.698	.000
Age	61.768	1	61.768	22.729	.000
Mile	3.985	1	3.985	1.466	.232
Error	127.725	47	2.718		
Total	13658.417	54			
Corrected Total	1404.546	53			

Dependent Variable: Price (1000 GBP)

a R Squared = .909 (Adjusted R Squared = .897)

## Model 10: Age + Mod + Mod . Age

Dependent Variable: Price (1000 GBP)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1292.291(a)	9	143.588	56.282	.000
Intercept	2147.345	1	2147.345	841.688	.000
Mod	270.552	4	67.638	26.512	.000
Age	160.470	1	160.470	62.899	.000
Mod * Age	19.455	4	4.864	1.906	.126
Error	112.254	44	2.551		
Total	13658.417	54			
Corrected Total	1404.546	53			

a R Squared = .920 (Adjusted R Squared = .904)

# Final Model: Age + Mod

		/			
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1272.836(a)	5	254.567	92.774	.000
Intercept	2949.842	1	2949.842	1075.032	.000
Mod	1014.703	4	253.676	92.449	.000
Age	167.368	1	167.368	60.995	.000
Error	131.710	48	2.744		
Total	13658.417	54			
Corrected Total	1404.546	53			

Dependent Variable: Price (1000 GBP)

a R Squared = .906 (Adjusted R Squared = .896)

### **Parameter Estimates**

Dependent Variable: Price (1000 GBP)

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	13.486	.684	19.720	.000	12.111	14.861
[Mod=0]	11.966	.726	16.482	.000	10.506	13.426
[Mod=1]	8.916	.948	9.401	.000	7.009	10.823
[Mod=2]	9.234	.674	13.709	.000	7.880	10.588
[Mod=3]	5.139	.599	8.582	.000	3.935	6.344
[Mod=4]	0(a)					
Age	-1.079	.138	-7.810	.000	-1.357	802

a This parameter is set to zero because it is redundant.

# Residuals



**Dependent Variable: Price (1000 GBP)** 

Model: Intercept + Mod + Age

# Q2. Oysters Data

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	198.407(a)	4	49.602	4.643	.012
Intercept	19028.281	1	19028.281	1780.979	.000
trt	198.407	4	49.602	4.643	.012
Error	160.263	15	10.684		
Total	19386.950	20			
Corrected Total	358.670	19			

a R Squared = .553 (Adjusted R Squared = .434)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	342.358(a)	1	342.358	377.793	.000
Intercept	6.466	1	6.466	7.135	.016
initialwt	342.358	1	342.358	377.793	.000
Error	16.312	18	.906		
Total	19386.950	20			
Corrected Total	358.670	19			

a R Squared = .955 (Adjusted R Squared = .952)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	354.447(a)	5	70.889	235.049	.000
Intercept	1.718	1	1.718	5.696	.032
trt	12.089	4	3.022	10.021	.000
initialwt	156.040	1	156.040	517.384	.000
Error	4.222	14	.302		
Total	19386.950	20			
Corrected Total	358.670	19			

a R Squared = .988 (Adjusted R Squared = .984)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	355.835(a)	9	39.537	139.510	.000
Intercept	.064	1	.064	.228	.644
trt	1.696	4	.424	1.496	.275
initialwt	68.529	1	68.529	241.809	.000
trt * initialwt	1.388	4	.347	1.225	.360
Error	2.834	10	.283		
Total	19386.950	20			
Corrected Total	358.670	19			

a R Squared = .992 (Adjusted R Squared = .985)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	354.447(a)	5	70.889	235.049	.000
Intercept	1.718	1	1.718	5.696	.032
trt	12.089	4	3.022	10.021	.000
initialwt	156.040	1	156.040	517.384	.000
Error	4.222	14	.302		
Total	19386.950	20			
Corrected Total	358.670	19			

a R Squared = .988 (Adjusted R Squared = .984)

Parameter Estimates									
					95% Confidence Interval				
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound			
Intercept	2.495	1.028	2.427	.029	.290	4.699			
[trt=1]	244	.577	424	.678	-1.481	.992			
[trt=2]	280	.493	569	.579	-1.337	.777			
[trt=3]	1.655	.429	3.853	.002	.734	2.576			
[trt=4]	1.107	.472	2.347	.034	.095	2.119			
[trt=5]	0(a)								
initialwt	1.083	.048	22.746	.000	.981	1.185			

a This parameter is set to zero because it is redundant.

### Dependent Variable: Final Weight (g)



Model: Intercept + trt + initialwt

## Q3. Oranges Data : Variety 1 data

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1346.889(a)	21	64.138	3.131	.016
Intercept	474.876	1	474.876	23.182	.000
day	113.659	5	22.732	1.110	.399
store	83.136	5	16.627	.812	.561
P1	336.892	1	336.892	16.446	.001
day * P1	106.533	5	21.307	1.040	.432
store * P1	82.077	5	16.415	.801	.567
Error	286.786	14	20.485		
Total	5391.693	36			
Corrected Total	1633.676	35			

Dependent Variable: Quantity Variety 1

a R Squared = .824 (Adjusted R Squared = .561)

Dependent Variable: Quantity Variety 1

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1264.813(a)	16	79.051	4.072	.002
Intercept	727.513	1	727.513	37.474	.000
day	98.792	5	19.758	1.018	.435
store	153.290	5	30.658	1.579	.214
P1	488.523	1	488.523	25.164	.000
day * P1	78.987	5	15.797	.814	.555
Error	368.863	19	19.414		
Total	5391.693	36			
Corrected Total	1633.676	35			

a R Squared = .774 (Adjusted R Squared = .584)

### Dependent Variable: Quantity Variety 1

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1185.825(a)	11	107.802	5.777	.000
Intercept	929.926	1	929.926	49.834	.000
day	456.054	5	91.211	4.888	.003
store	238.695	5	47.739	2.558	.054
P1	622.008	1	622.008	33.333	.000
Error	447.850	24	18.660		
Total	5391.693	36			
Corrected Total	1633.676	35			

a R Squared = .726 (Adjusted R Squared = .600)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	947.131(a)	6	157.855	6.668	.000
Intercept	1094.386	1	1094.386	46.227	.000
day	430.538	5	86.108	3.637	.011
P1	696.733	1	696.733	29.430	.000
Error	686.545	29	23.674		
Total	5391.693	36			
Corrected Total	1633.676	35			

Dependent Variable: Quantity Variety 1

a R Squared = .580 (Adjusted R Squared = .493)

Dependent Variable: Quantity Variety 1

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1111.523(a)	11	101.048	4.644	.001
Intercept	854.684	1	854.684	39.284	.000
P1	554.786	1	554.786	25.500	.000
day	201.172	5	40.234	1.849	.141
day * P1	164.392	5	32.878	1.511	.224
Error	522.153	24	21.756		
Total	5391.693	36			
Corrected Total	1633.676	35			

a R Squared = .680 (Adjusted R Squared = .534)

Dependent Variable: Quantity Variety 1

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	516.592(a)	1	516.592	15.723	.000
Intercept	882.193	1	882.193	26.851	.000
P1	516.592	1	516.592	15.723	.000
Error	1117.084	34	32.855		
Total	5391.693	36			
Corrected Total	1633.676	35			

a R Squared = .316 (Adjusted R Squared = .296)

# Q2. Oranges Data : Variety 2 data

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2474.507(a)	21	117.834	5.984	.001
Intercept	856.079	1	856.079	43.471	.000
store	189.615	5	37.923	1.926	.154
day	321.845	5	64.369	3.269	.037
P2	475.965	1	475.965	24.169	.000
day * P2	277.857	5	55.571	2.822	.058
store * P2	188.695	5	37.739	1.916	.155
Error	275.701	14	19.693		
Total	7155.720	36			
Corrected Total	2750.208	35			

Dependent Variable: Quantity Variety 2

a R Squared = .900 (Adjusted R Squared = .749)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2285.812(a)	16	142.863	5.845	.000
Intercept	986.830	1	986.830	40.375	.000
store	381.265	5	76.253	3.120	.032
day	438.673	5	87.735	3.590	.019
P2	587.279	1	587.279	24.028	.000
day * P2	326.468	5	65.294	2.671	.054
Error	464.396	19	24.442		
Total	7155.720	36			
Corrected Total	2750.208	35			

Dependent Variable: Quantity Variety 2

a R Squared = .831 (Adjusted R Squared = .689)

#### Dependent Variable: Quantity Variety 2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1959.344(a)	11	178.122	5.405	.000
Intercept	1682.016	1	1682.016	51.043	.000
store	309.879	5	61.976	1.881	.135
day	773.130	5	154.626	4.692	.004
P2	1001.024	1	1001.024	30.378	.000
Error	790.864	24	32.953		
Total	7155.720	36			
Corrected Total	2750.208	35			

a R Squared = .712 (Adjusted R Squared = .581)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1649.465(a)	6	274.911	7.243	.000
Intercept	1529.311	1	1529.311	40.291	.000
day	763.905	5	152.781	4.025	.007
P2	862.204	1	862.204	22.715	.000
Error	1100.743	29	37.957		
Total	7155.720	36			
Corrected Total	2750.208	35			

Dependent Variable: Quantity Variety 2

a R Squared = .600 (Adjusted R Squared = .517)

Dependent Variable: Quantity Variety 2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1904.547(a)	11	173.141	4.914	.001
Intercept	894.399	1	894.399	25.383	.000
day	371.719	5	74.344	2.110	.099
P2	510.672	1	510.672	14.493	.001
day * P2	255.082	5	51.016	1.448	.244
Error	845.661	24	35.236		
Total	7155.720	36			
Corrected Total	2750.208	35			

a R Squared = .693 (Adjusted R Squared = .552)

### Dependent Variable: Quantity Variety 2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	885.560(a)	1	885.560	16.147	.000
Intercept	1600.103	1	1600.103	29.176	.000
P2	885.560	1	885.560	16.147	.000
Error	1864.648	34	54.843		
Total	7155.720	36			
Corrected Total	2750.208	35			

a R Squared = .322 (Adjusted R Squared = .302)

# Q4. Cotton Data

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	31.361(a)	7	4.480	106.169	.000
Intercept	.008	1	.008	.188	.667
variety	.057	1	.057	1.349	.252
spacing	.026	1	.026	.611	.439
bollwt	8.210	1	8.210	194.569	.000
variety * spacing	.003	1	.003	.075	.785
variety * bollwt	.328	1	.328	7.768	.008
spacing * bollwt	.001	1	.001	.014	.907
variety * spacing * bollwt	4.13E-008	1	4.13E-008	.000	.999
Error	1.730	41	.042		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .948 (Adjusted R Squared = .939)

	Type III Sum				
Source	of Squares	df	Mean Square	F	Sig.
Corrected Model	31.361(a)	6	5.227	126.884	.000
Intercept	.009	1	.009	.221	.641
variety	.078	1	.078	1.889	.177
spacing	.031	1	.031	.749	.392
bollwt	8.718	1	8.718	211.637	.000
variety * spacing	.032	1	.032	.769	.386
variety * bollwt	.448	1	.448	10.881	.002
spacing * bollwt	.001	1	.001	.015	.904
Error	1.730	42	.041		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .948 (Adjusted R Squared = .940)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	31.360(a)	5	6.272	155.829	.000
Intercept	.015	1	.015	.377	.542
variety	.091	1	.091	2.250	.141
spacing	.390	1	.390	9.691	.003
bollwt	11.649	1	11.649	289.413	.000
variety * spacing	.079	1	.079	1.953	.169
variety * bollwt	.467	1	.467	11.606	.001
Error	1.731	43	.040		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .948 (Adjusted R Squared = .942)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	31.281(a)	4	7.820	190.179	.000
Intercept	.011	1	.011	.258	.614
variety	.079	1	.079	1.929	.172
spacing	.375	1	.375	9.123	.004
bollwt	11.573	1	11.573	281.444	.000
variety * bollwt	.482	1	.482	11.717	.001
Error	1.809	44	.041		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .945 (Adjusted R Squared = .940)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	30.906(a)	3	10.302	212.222	.000
Intercept	.022	1	.022	.459	.502
bollwt	11.452	1	11.452	235.905	.000
variety	.109	1	.109	2.241	.141
variety * bollwt	.573	1	.573	11.810	.001
Error	2.184	45	.049		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .934 (Adjusted R Squared = .930)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	30.799(a)	3	10.266	201.645	.000
Intercept	.003	1	.003	.065	.800
variety	1.197	1	1.197	23.517	.000
spacing	.467	1	.467	9.165	.004
bollwt	11.572	1	11.572	227.282	.000
Error	2.291	45	.051		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .931 (Adjusted R Squared = .926)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	30.333(a)	2	15.166	252.979	.000
Intercept	.000	1	.000	.007	.934
variety	1.264	1	1.264	21.076	.000
bollwt	11.434	1	11.434	190.724	.000
Error	2.758	46	.060		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .917 (Adjusted R Squared = .913)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	29.069(a)	1	29.069	339.755	.000
Intercept	.743	1	.743	8.689	.005
bollwt	29.069	1	29.069	339.755	.000
Error	4.021	47	.086		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .878 (Adjusted R Squared = .876)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	31.281(a)	4	7.820	190.179	.000
Intercept	.011	1	.011	.258	.614
variety	.079	1	.079	1.929	.172
spacing	.375	1	.375	9.123	.004
bollwt	11.573	1	11.573	281.444	.000
variety * bollwt	.482	1	.482	11.717	.001
Error	1.809	44	.041		
Total	187.560	49			
Corrected Total	33.091	48			

a R Squared = .945 (Adjusted R Squared = .940)

### Parameter Estimates

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	.006	.125	.051	.959	245	.258
[variety=37]	312	.224	-1.389	.172	764	.141
[variety=213]	0(a)					
[spacing=30]	.185	.061	3.020	.004	.062	.308
[spacing=40]	0(a)					
bollwt	.240	.025	9.414	.000	.188	.291
[variety=37] * bollwt	.124	.036	3.423	.001	.051	.196
[variety=213] * bollwt	0(a)					

Dependent Variable: Total Lint Weight (g)

a This parameter is set to zero because it is redundant.



## Dependent Variable: Total Lint Weight (g)

Model: Intercept + variety + spacing + bollwt + bollwt .variety

# Q5. Doses Data

Courses	Type III Sum	-14	Maan Cruana	F	Circ
Source	or Squares	ar	Mean Square	F	Sig.
Corrected Model	994.042(a)	17	58.473	4.683	.033
Intercept	72490.042	1	72490.042	5805.654	.000
bloc	538.792	3	179.597	14.384	.004
type	12.042	1	12.042	.964	.364
dose	121.583	2	60.792	4.869	.055
bloc * type	47.458	3	15.819	1.267	.367
bloc * dose	130.083	6	21.681	1.736	.260
type * dose	144.083	2	72.042	5.770	.040
Error	74.917	6	12.486		
Total	73559.000	24			
Corrected Total	1068.958	23			

a R Squared = .930 (Adjusted R Squared = .731)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	946.583(a)	14	67.613	4.973	.010
Intercept	72490.042	1	72490.042	5331.239	.000
bloc	538.792	3	179.597	13.208	.001
type	12.042	1	12.042	.886	.371
dose	121.583	2	60.792	4.471	.045
bloc * dose	130.083	6	21.681	1.594	.254
type * dose	144.083	2	72.042	5.298	.030
Error	122.375	9	13.597		
Total	73559.000	24			
Corrected Total	1068.958	23			

a R Squared = .886 (Adjusted R Squared = .707)

### Dependent Variable: Response

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	816.500(a)	8	102.063	6.064	.001
Intercept	72490.042	1	72490.042	4307.050	.000
bloc	538.792	3	179.597	10.671	.001
type	12.042	1	12.042	.715	.411
dose	121.583	2	60.792	3.612	.052
type * dose	144.083	2	72.042	4.280	.034
Error	252.458	15	16.831		
Total	73559.000	24			
Corrected Total	1068.958	23			

a R Squared = .764 (Adjusted R Squared = .638)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	672.417(a)	6	112.069	4.804	.005
Intercept	72490.042	1	72490.042	3107.695	.000
bloc	538.792	3	179.597	7.699	.002
type	12.042	1	12.042	.516	.482
dose	121.583	2	60.792	2.606	.103
Error	396.542	17	23.326		
Total	73559.000	24			
Corrected Total	1068.958	23			

a R Squared = .629 (Adjusted R Squared = .498)

# **Final Model**

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	816.500(a)	8	102.063	6.064	.001
Intercept	72490.042	1	72490.042	4307.050	.000
bloc	538.792	3	179.597	10.671	.001
dose	121.583	2	60.792	3.612	.052
type	12.042	1	12.042	.715	.411
dose * type	144.083	2	72.042	4.280	.034
Error	252.458	15	16.831		
Total	73559.000	24			
Corrected Total	1068.958	23			

a R Squared = .764 (Adjusted R Squared = .638)

Parameter Estimates

					95% Confidence Interval	
Parameter	В	Std. Error	t	Sig.	Lower Bound	Upper Bound
Intercept	62.042	2.512	24.696	.000	56.687	67.396
[bloc=1]	7.667	2.369	3.237	.006	2.618	12.715
[bloc=2]	-3.500	2.369	-1.478	.160	-8.549	1.549
[bloc=3]	-4.333	2.369	-1.830	.087	-9.382	.715
[bloc=4]	0(a)					
[dose=1]	-11.250	2.901	-3.878	.001	-17.433	-5.067
[dose=10]	-7.750	2.901	-2.672	.017	-13.933	-1.567
[dose=100]	0(a)					
[type=1]	-8.000	2.901	-2.758	.015	-14.183	-1.817
[type=2]	0(a)					
[dose=1] * [type=1]	11.750	4.103	2.864	.012	3.006	20.494
[dose=1] * [type=2]	0(a)					
[dose=10] * [type=1]	8.000	4.103	1.950	.070	744	16.744
[dose=10] * [type=2]	0(a)					
[dose=100] * [type=1]	0(a)					
[dose=100] * [type=2]	0(a)				•	

a This parameter is set to zero because it is redundant.



# Dependent Variable: Response

Model: Intercept + bloc + dose + type + dose \* type