MATH 204 - ASSIGNMENT 3: SOLUTIONS

1. For each centre separately, we test the hypothesis

$$H_0 \ : \ p_1 = p_2 = p_3 = p_4 = \frac{1}{4}$$

against the alternative hypothesis that H_0 is not true. In the formula

$$X^2 = \sum_{i=1}^{k} \frac{\left(n_i - np_i^{(0)}\right)^2}{np_i^{(0)}} = \sum_{i=1}^{k} \frac{(\text{Observed Count in Cell } i - \text{Expected Count in Cell } i)^2}{\text{Expected Count in Cell } i}$$

with $k = 4$, the fitted values are therefore $np_i^{(0)} = 24/4 = 6$. The boundary of the rejection region is, from tables, $C_R = \text{Chisq}_{0.05}(k-1) = \text{Chisq}_{0.05}(3) = 7.81$.

	Arm					
Centre	1	2	3	4	X^2	Reject H_0
1	6	8	5	5	1.00	No
2	6	9	3	6	3.00	No
3	7	10	1	6	7.00	No

Thus, despite the apparent imbalances in the counts, there is insufficient evidence to reject H_0.

6 Marks

In this case the expected counts are all equal to six, so the usual guideline that the expected counts need to be at least five is met.

2 Marks

2. (a) For this hypothesis, you need to use the **Wilcoxon signed ranks** test for **paired data**, as the T_4 and T_8 measurements are made on the same experimental subjects. Here $n = 20$, and we are looking for a **higher** T_8 count than T_4 count, so the hypotheses of interest are

H_0 : No change between first and second measurements
H_a : Significant **increase** between first and second measurements

Thus
- a **large rank sum** for the **negative ranks** T_-, or
- a **small rank sum** for the **positive ranks** T_+,

for the differences

$$x_i = T_{4i} - T_{8i}$$

implies that we should **reject** H_0. By direct calculation (or using SPSS) we have that

$$T_+ = 148 \qquad T_- = 62$$

(the signs are reversed compared to SPSS as SPSS computes the difference $x_i = T_{8i} - T_{4i}$).

From Tables, for $n = 20$, we see that for the required **one-tailed** test, the $\alpha = 0.05$ critical value is 60. That is, if $T_+ \leq 60$, we would reject H_0 in favour of H_1. But here $T_+ = 148 > 60$, so we **do not reject** H_0. This is confirmed by the asymptotic test performed by SPSS (see SPSS output). In the output, the p-value for the **two-tailed** test is quoted; to get the p-value for the one-tailed test, we simply divide by two to get $0.108/2 = 0.0502$, so again the test does not reject H_0. **However, this test is for the alternative hypothesis**

H_a : Significant **decrease** between first and second measurements

which is not the one of interest. To get the p-value for the H_a of interest, we need to compute the p-value as

$$p = 1 - 0.108/2 = 0.948.$$

6 Marks

(a) For this hypothesis, you need to use the **Mann-Whitney-Wilcoxons** test for independent samples, as the T_4/T_8 ratios are computed independently on different experimental subjects. Here $n = 20$, and we are looking for a **different** T_4/T_8 ratio, so the hypotheses of interest are

$$\begin{aligned} H_0 &: \eta_1 = \eta_2 \\ H_1 &: \eta_1 \neq \eta_2 \end{aligned}$$

Thus we need the rank sum R_2 to satisfy $R_2 \leq T_L$ or $R_2 \geq T_U$. The table does not give T_L and T_U for $n = 20$, but the SPSS output (see SPSS output) does give the exact p-value.

By direct calculation on the T_4/T_8 **ratios**, or from SPSS , we see that $R_2 = 312$, and that $Z = -2.651$. In the output, the asymptotic (Normal approximation) p-value is quoted as 0.008, and the exact p-value is given as 0.007. Hence H_0 is rejected in favour of H_1.

The output also indicates the direction of the change; the rank sum for Group 2 is **higher** (508) than that for Group 1 (312), indicating that the T_4/T_8 ratio is **higher** in Group 2.

6 Marks

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
Group 1 T8 - Group 1 T4	Negative Ranks	12[a]	12.33	148.00
	Positive Ranks	8[b]	7.75	62.00
	Ties	0[c]		
	Total	20		

a. Group 1 T8 < Group 1 T4

b. Group 1 T8 > Group 1 T4

c. Group 1 T8 = Group 1 T4

Test Statistics[b]

	Group 1 T8 - Group 1 T4
Z	-1.605[a]
Asymp. Sig. (2-tailed)	.108

a. Based on positive ranks.

b. Wilcoxon Signed Ranks Test

Mann-Whitney Test

Ranks

	Group	N	Mean Rank	Sum of Ranks
T4/T8 Ratio	Hodgkin's	20	15.60	312.00
	Non-Hodgkin's	20	25.40	508.00
	Total	40		

Test Statistics[b]

	T4/T8 Ratio
Mann-Whitney U	102.000
Wilcoxon W	312.000
Z	-2.651
Asymp. Sig. (2-tailed)	.008
Exact Sig. [2*(1-tailed Sig.)]	.007[a]

a. Not corrected for ties.

b. Grouping Variable: Group