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STATISTICAL METHODS IN **BIOINFORMATICS**

TEN THINGS YOU NEED TO KNOW ABOUT AND UNDERSTAND

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Randomness and Uncertainty

By carrying out experiments to measure some phenomenon, we gather relevant but imperfect information about the real-world.

We may observe data that leads us to suspect that some important feature may be present in the data generation model.

We only obtain a finite number of measurements, so our information is incomplete; we have some remaining uncertainty about the state of the world, and it is likely that what we have observed is not an accurate picture.

Key Question: Is the observed effect genuine, or due purely to chance ?

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	- 20	СТСТА	AATTG	ТСТАС	ATGT	CCGC	GACC	САТА	ТАСТТС	TACGCA	TTAAGTAG
		TTGCC	ЭСАТС	ССССТ	GCTC	GGCG	AACG	ACTG	CGCAAG	ССТАТА	Т G T G T A A T
		AAATI	гттсс	AGATA	GGCA	A T G G	GGAG	СТССС	СТТТБА	GTCACG	АСАСТСТА
		AGAAT	F A G C G	GTAAG	ACCA	GТАТ	GGGT	GTTC	ТТАТСС	GAGTCA	GCGTCTTT
		TATGA	ACCAG	ACAGG	ACCC	A T G G	GGTA	CCAA	ТАТТТА	АССТТА	ТАСАСТСА
	- 15	CATAT	гттаа	GΤΑΤΤ	GAGT	ТТАС	ССАА	GGGG	AAGATT	ААСТСА	G T G A A C T T
		GGTAA	AGCA	TGCAA	CGGC	ТААА	GAGT	GACGO	ССБАТТ	TGCAAC	ААТССТАТ
		GATAT	гттаа	CGATC	TCGG	тссс	CCGC	тдтт	AGTGAA	АСТАСТ	ТАААССТА
		TGTCA	A G A A A	AGGCC	ACGG	CGAC	ТССС	САТТ	АССТАБ	GGAACC	АТТСАССА
		CAAAA	АТТСА	GCAAT	CGTG	GACA	GTTT	тссто	GGGCGG	T T G A G A	ААССТТСС
~	- 1	СТАТТ	гстсс	ТАСТС	ТТСА	ААСС	CAGG	ATGG	ACAGAA	GCCCAG	Т G C G G C T A
		TGCAT	ГАСТБ	GGGAA	СТАА	GCGC	GATC	TACA	ТАТТТА	AACGAG	AGATTAAG
		CAGTO	GTGAT	СТААС	GTCC	GТАС	ТGСG	TCGG	зссстс	CAAGCA	ТАТБББТБ
		ТАСТА	ACGCT	GACAA	AACA	CGCA	TGAC	GGGG	ТТААСС	СССТСТ	тсасттст
		ATTTA	AGGCG	GGAAC	TCGA	GGAT	тстб	GAAGO	ЗСССТС	GATCAT	TGCCAGAG
	- <u>م</u>	ATTCC	GCGAC	GCCGG	GGCG	сссс	TATG	TAGC	ттттсс	GTTTAG	CCCCGAAC
		AGGGG	GAAG	ттссс	GTAA	тстс	AACT	ACAC	ТААТСА	СТТАТА	GTACAAGA
		ATCAT	ГАТАС	GТТСТ	ATAG	АССТ	ATGA	GGAC	ССТТАС	САТТСА	ТАТТТААА
		AGATO	CAGT	ТТАСТ	СТТТ	CAGT	TGCA	CGGG	TAGAGC	AGCGTG	GCGGTAAG
		AAACC	ССАТС	AATGT	ACAC	АТТА	ТАТТ	ТААТ	ССТБАТ	GATCAC	сстттссс
	L			1		1				1	1
		0		10		20		30		40	50

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Models Monte Carlo

Statistical **DNA Sequences** Methods in Bioinformatics Randomness

and Uncertainty	- 50	CTCTAA TTGCGC	T T G T C T A G A T C C G C C T	G A T G T C C G C G A I G C T C G G C G A A	CCCATATAGTT CGACTGCGCAA	CTACGCATT. GCCTATATG	A A G T A G T G T A A T
Probability		ΑΑΑΤΤΤ	T G C A G A T A	G G C A A T G G G G	AGCTCGCTTTG	AGTCACGAC	АСТСТА
Statistical Summary	_ <u>م</u>	AGAATA TATGAC CATATT	G C G G T A A G C A G A C A G G T A A G T A T T	GACCAGTATGG GACCCATGGGG GAGTTTAGCC	G T G T T C T T A T G T A C C A A T A T T T A A G G G G A A G A T	GGAGTCAGC	GTCTTT CAGTCA GAACTT
Hypothesis Testing	-	GGTAAA GATATT	GCATGCAA TAACGATC	CGGCTAAAGA CTCGGTCCCCC	GTGACGCCGAT GCTGTTAGTGA	T T G C A A C A A A A C T A G T T A	T G C T A T A A C G T A
Multiple Testing Corrections	× 6 -	TGTCAG CAAAAT CTATTG	А А А А G G C C Т C A G C A A T Т G G T A C T C	С А С G G C G A C T G Г C G T G G A C A G T С T T C A A A C C C A	C G C A T T A C C T A T T T C G T G G G C G G G A T G G A C A G A	G G G A A C C A T [*] G T T G A G A A A A G C C C A G T G	TCACCA GGTTCC CGGCTA
Resampling Procedures		T G C A T A C A G T G T	C T G G G G A A G A T C T A A C	\	T C T A C A T A T T T C G T C G G G C C C T	A A A C G A G A G A G A G A G A G A G A G A	A T T A A G T G G G T G
Regression and Classification	– <u>م</u>		G C T G A C A A G C G G G G A A C G A C G C C G G	\	ACGGGGTTAAC TGGAAGGGCGT	GCGCTCTTC. GGATCATTG CGTTTAGCC	ACTTCT CCAGAG
Clustering		AGGGCG	A A G T T C G C T A G G T T C T	С	CTACACTAATC GAGGACCCTTA	ACTTATAGT. CCATTG ATA	A C A A G A T T T A A A
Hidden Markov Models		AGATCC AAACCG	А G T T T A C T А T C A A T G T	CTTTCAGTTG ACACATTATA	CACGGGTAGAG TTTAATCCTGA	CAGCGTGGC TGATCACCC	GGTAAG TTTCCC
Monte Carlo		0	10	20	30	40	50

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Randomness and Uncertainty

Randomness and Uncertainty

As $n \to \infty$

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$\overline{x} \longrightarrow \mu$ Population mean $s^2 \longrightarrow \sigma^2$ Population variance

 $\frac{s}{\sqrt{n}} \longrightarrow 0$

Thus the standard error is a measure of uncertainty in our reported mean.

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Statistical Randomness and Uncertainty Methods in Bioinformatics

We collect *n* observations.

- if *n* is large, we have learned a considerable amount,
- if *n* is small, we may have learned little.

We should reflect the sample size in our summary of variability.

Sample mean: $\bar{x} = (x_1 + x_2 + ... + x_n)/n = \frac{1}{n} \sum_{i=1}^n x_i$

Sample variance : $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2$

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Standard Error: s.e. = $\frac{s}{\sqrt{n}}$

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Probability Methods in Bioinformatics

Probability theory provides the underpinning of all of statistics.

It gives a formal mathematical framework for the assessment of uncertainty in experimental outcomes.

The key components are

- the list of possible outcomes; the sample space
- interesting collections of those outcomes; events
- a way of assigning a numerical value (probability) to each collection.

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Probability: The Axioms

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Let E and F denote two events, that is, two collections of outcomes of interest. Let Ω denote the entire collection of possible outcomes.

Let P denote the probability function, and P(E) denote the probability that E occurs, that is, the probability that the actual outcome (when we observe it) is one of the collection called E.

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Probability: The Axioms

Example

In microarray experiments, gene expression for a large number of genes is measured using fluorescence imaging. The intensity of fluorescence is measured in a spot corresponding to a gene or EST.

The intensity measurement scale typically runs from $2^6 = 64$ to $2^{16} = 65536$. We might wish to assess the chance that the intensity measurement is between $2^8 = 256$ and $2^{10} = 1024$, as this corresponds to biologically significant up-regulation. Thus

• $\Omega = \{64, 65, \dots, 65536\}$

• $E = \{256, 267, \dots, 1024\}$

But what is P(E) ?

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Probability Assignment

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These mathematical rules do not tell us how to assign P(E).

Classical Approach

Consider all outcomes equally likely

Let *E* contain n_F sample outcomes, and let Ω contain *n* outcomes. Then

 $P(E) = \frac{n_E}{n}$

Frequentist Approach

Consider the relative frequency with which *E* occurs. In *n* hypothetical repeats of the experiment, suppose that *E* occurs n_E times. Then, if *n* is large

 $P(E) = \lim_{n \to \infty} \frac{n_E}{n}$

Subjective Approach

Consider your personal degree of belief that *E* occurs.

P must exhibit the following properties:

 $\mid P(E) \mid est \mid P(E) \mid est \mid P(E) \mid P(E) \mid est \mid P(E) \mid est \mid est \mid est \mid P(E) \mid est \mid es$

 $0 \leq P(E) \leq 1$

II The probability that the actual outcome is in the collection Ω is 1.

 $P(\Omega) = 1.$

III If E and F have no common elements, then the probability the actual outcome lies within either E or F is the sum of the probabilities of E and F.

P(E or F) = P(E) + P(F)

Theorem

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Statistical **Probability Assignment** Methods in Bioinformatics

• Easy Case : Coins - Let E be "Heads"

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What is P(E) ?

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What is P(E) ?





Hypothesis Testing

Hypothesis (significance) testing is the formal statistical procedure for assessing the adequacy of a proposed theoretical model.

It compares the predicted behaviour of a data-related quantity (or statistic) under a proposed model with its observed behaviour in the data sample.

If the predicted and observed behaviour do not coincide, then the proposed model is deemed inadequate; if the observed behaviour is surprising under the proposed model, then proposed model can be rejected.

The statistical formulation formalizes the quantification of evidence.

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Statistical Summaries Statistical Methods in Bioinformatics

results of experimentation.

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- - correlations
 - Graphical

Numerical

one variable: histograms, boxplots, density, cumulative

sample means, variances, standard errors, skewness

Use of numerical or graphical methods to summarize the

two variables: scatterplot

guantiles, median, min, max

many variables: scatterplot matrices

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Hypothesis Testing

Formally, two models are compared:

: The Null Model H₀

 H_1 : The Alternative Model

Typically, the null model is a simpler model that is of less scientific interest.

The objective of hypothesis testing is to see whether the model described by H_0 is adequate, or whether it must be rejected in favour of the more complicated model.

Statistical Hypothesis Testing Methods in Bioinformatics

TEST STATISTIC

NULL DISTRIBUTION

• P-VALUE, denoted p.

CRITICAL VALUE(S)

SIGNIFICANCE LEVEL, denoted α

namely

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Hypothesis Testing: Significance Level and **Critical Value**

There are five crucial components to a hypothesis test,

What constitutes a "surprising" result ?

This is specified by the **significance level**, α , chosen by considering an acceptable value

 $\alpha = P[T < t_{CR} \text{ or } T > t_{CR}, \text{ given that } H_0 \text{ is TRUE}]$

that is, the probability that the predicted test statistic is more extreme than some cut-off value t_{CB} , if the null model is in fact correct.

 α essentially quantifies how large or small we predict test statistic T to be by implicitly defining the critical value, t_{CB} . α is the maximum allowable false positive probability in the test of H_0 .

Typically, a value of $\alpha = 0.05$ or $\alpha = 0.01$ is selected.

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Hypothesis Testing : Strategy Bioinformatics

For a data sample x_1, \ldots, x_n ;

- 1. consider a pair of competing **hypotheses**, H_0 and H_1
- 2. define a suitable test statistic T.
- 3. assume that H_0 is true, and compute the predicted probability distribution of T; this is the **null distribution**
- 4. compute the **observed** value of T, $t = T(x_1, ..., x_n)$; this is the test statistic
- 5. assess whether *t* is a surprising observation the distribution of T, that is whether t is in a region of low probability. If it is surprising, there is evidence to reject H_0 ; if it is not surprising, we **cannot reject** H_0

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Hypothesis Testing: p-value

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The *p*-value of a significance test is the probability *p* defined by

```
p = P[T < t \text{ or } T > t, \text{ given that } H_0 \text{ is TRUE}]
```

that is, the probability that the predicted test statistic is more extreme than the observed test statistic t, if the null model is in fact correct.

p quantifies the surprisingness of the result, that is, if p is small, the result would be surprising if the null model was in fact true.

Statistical Hypothesis Testing: Example Methods in Bioinformatics

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Example

In microarray analysis, replicate arrays yield expression measurements for each of two tissue samples for thousands of genes.

We might have measurements of the expression levels for a single gene

- x_1, \ldots, x_n for tissue A
- y_1, \ldots, y_m for tissue B

and it is of interest to consider whether there is differential expression between to the two tissues.

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Hypothesis Testing: Example

two samples.

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The null distribution of this test statistic is centered at zero:

Example (Continued)

a large positive or negative value of t leads us to reject H_0 , and conclude that there is in fact differential expression.

In this formula s_x^2 and s_y^2 are the sample variances from the

In fact, any such test statistic can be used. However, the null distribution may not be easy to compute, even if simplifying assumptions are made.

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Hypothesis Testing: Example Bioinformatics

Example (Continued)

We consider the two data samples as being generated from populations with means μ_X and μ_Y respectively.

 H_0 : $\mu_X = \mu_Y$

 H_1 : $\mu_X \neq \mu_Y$

 $t = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}}$

Then a test of differential expression is a test of the hypotheses

for which an appropriate test statistic is

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which is a standardized difference in observed means.

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Hypothesis Testing: Confidence Intervals

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Hypothesis tests are closely related to confidence intervals, that are regions which capture the estimated value and some measure of uncertainty about a parameter of interest.

In estimation of a parameter θ , a 95 % Confidence Interval is the set of possible constants c for which the null hypothesis

 $H_0: \theta = c$

is **NOT** rejected given the data being studied at the $\alpha = 0.05$ significance level

More loosely (but incorrectly), a 95 % confidence interval can be interpreted as an interval within which the "true value" of θ lies with probability 0.95.

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Hypothesis Testing: Types of Test

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Tests for Parameters

- z-tests, t-tests, Wald tests for coefficients
- Tests for Models
 - ANOVA, F-test
- Goodness of Fit Tests
- Non-parametric Tests
 - Wilcoxon, Kolmogorov-Smirnov

The multiple testing corrections are used when several independent statistical tests are being performed

In order to avoid a lot of spurious positives, when the null

hypothesis is rejected when it is actually true, the α value

needs to be lowered to account for the number of

simultaneously, as in microarray analyses.

Multiple Testing Corrections

They are necessary because, while a given significance level α may be appropriate for each individual comparison, it is not appropriate for the set of all comparisons. The idea of a multiple testing correction is to control the rate of false positive results when many significance tests are

being carried out.

comparisons being performed.

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- - Chi-squared test

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Multiple Testing Corrections

When carrying out K significance tests, the simplest and most conservative approach is the Bonferroni correction, which sets the significance level for each test

$$\alpha_B(K) = \frac{\alpha}{K}$$

where α is a significance level that would be assumed for a single test.

This correction is usually too extreme, and is somewhat of an over-correction. More advanced corrections have been proposed that have better testing properties.

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Hypothesis Testing: Example

Example

In a genomic sequence alignment problem, the BLAST tool is used to assess the alignments by computing a

- test statistic (an alignment score)
- the null distribution (based on large sample approximations)
- a p-value and an E-value, where E is the expected number of hits that a database search would reveal for which the test statistic would be more extreme than the observed test statistic.

In fact

$$\mathsf{E} = \mathsf{N}\mathsf{K} e^{-\lambda y_{\mathsf{max}}}$$

for certain parameters K and λ estimated from the database.

Multiple Testing Corrections: FDR Methods in Bioinformatics

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An alternative approach to multiple testing procedures is based on the assessment of the False Discovery Rate (FDR).

Suppose that K tests are to be carried out. Then

Number of False Test Rejections $FDR = \frac{1}{\text{Total Number of Test Rejections}}$

if the denominator is greater than zero.

In any practical example, we do not know the numerator. However, we can design test procedures that guarantee to limit the FDR to be below some fixed value (say 10 %).

The central idea of permutation tests uses rearrangements of the data (permutations) to compute the null distribution in a hypothesis test.

- Often a null hypothesis imposes some form of symmetry on the observed data.
- Using permutation procedures, the sampling distribution of the test statistic under the null hypothesis is computed by forming all (or many) of the permutations that respect the symmetry calculating the test statistic for each.
- These pseudo-test statistics are used to form the null distribution, and hence compute test p-values etc.

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Bootstrap and Randomization Procedures

Resampling techniques such as the bootstrap standard error, confidence intervals, and distributions for any statistic.

By resampling observations from the observed data, the process of sampling observations from the population is mimicked.

In bootstrap resampling,

- B new samples, of the same size as the observed data, are drawn with replacement from the observed data.
- The statistic of interest is first calculated using the observed data and then recalculated using each of the new samples, yielding a bootstrap distribution.
- The resulting replicates are used to calculate the bootstrap estimates of bias, mean, and standard error for the statistic.
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Permutation tests: Example

Example

In a microarray analysis, suppose 10 and 8 measurements for two tissue samples are available, with data means

> 5.149 and 15.033.

We wish to test whether there is a mean difference using permutation methods. There are

$$\binom{18}{8} = 43758$$

possible relabellings of the samples consistent with the null hypothesis

$$H_0: \mu_X = \mu_Y$$

so we can compute the exact null distribution by computing the chosen test statistic for each possible relabelling.

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Permutation tests: Example Bioinformatics

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Example (Continued) Observed Test statistic:

 $t = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = -7.99$

Regression and Classification

a function of a collection of other variables.

• response variable, Y

• predictors, X_1, \ldots, X_d

additional random component.

Permutation test statistic: computed by re-allocating at random 10 observations from 18 to tissue A and remaining 8 to tissue B, and then recomputing t, for all possible re-allocations.

Regression involves modelling the variability in a variable as

We fit a model for Y as a function of the X variables that

captures the systematic component of variation, with an

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Clear evidence to reject H_0 .

Regression and Classification

Randomness

Two cases

- Normal Linear Regression
 - continuous data
 - normal (Gaussian) assumption for random variability
- Generalized Linear Model
 - categorical data
 - needs other probabilistic model for random variability

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Permutation tests: Example

Example (Continued)

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Uncertainty Probability

If the response variable is categorical, we might be interested in performing a classification exercise

- Y = 0 or Y = 1 correspond to two classes (healthy/affected, normal/tumour)
- access to some labelled cases
- · case status known for some individuals, unknown for others
- · objective is to label the unknown cases

Labelled cases are known as training data

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Regression and Classification: Example

Example

Microarray data provide large amounts of information on gene expression. An objective is to uncover a signature expression profile that allows discrimination between tissue types.

In cancer studies it may be useful to identify characteristic signatures of different tumour types; such studies are statistically challenging

- Usually many thousands of genes
- Usually very few cases
- In a famous example (Golub et al., 1999)
 - ALL/AML tumours
 - ► ~ 7000 genes

72 cases

Statistical **Regression and Classification** Methods in Bioinformatics

Discrimination

Links to other forms of multivariate analysis.

Principal Components Analysis (PCA)

· Least-squares and Partial Least Squares

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Clustering

Hierarchical clustering results can be displayed using a dendrogram



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Statistical Clustering Methods in Bioinformatics

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Cluster analysis involves searching for groups (clusters) in the data, in such a way that objects belonging to the same cluster resemble each other, whereas objects in different clusters are dissimilar.

There are two basic sorts of algorithm

- Partitioning algorithms
- Hierarchical algorithms

Both use the basic concept of similarity, but have different rules for defining the subgroups.

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Hidden Markov Models

In a biological sequence analysis, suppose that the character at a given sequence position can be classified as being in a **region** of one of *H* possible regions.

Within a region labelled by h the observed nucleotide sequence follows a Markov chain

- A Markov chain is a particular stochastic process model that captures the dependencies between adjacent positions or regions.
- The chain is specified by two quantities
 - ▶ a transition matrix *P* that specifies the probabilities that one character is followed by another in the sequence,
 - a probability vector p_0 that specifies the distribution of the initial character

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To complete the specification, we assume that there is a sequence in parallel to the observed sequence, comprising region label random variables H_1, H_2, \dots which itself follows a Markov chain governed by transition matrix P_{θ} with $(i, j)^{\text{th}}$ element

 $P(\text{Region type } i \text{ at time } t \rightarrow \text{Region type } j \text{ at time } t+1)$

The key difference is that the $\{H_t\}$ sequence is unobserved or hidden.

Our objective is to uncover the hidden sequence of regions or states from the observed data.

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Hidden Markov Models Bioinformatics

Randomness and Uncertainty

We use special algorithms to estimate the parameters and the hidden state sequence

- forward/backward algorithms
- Viterbi algorithm
- Baum-Welch algorithm

Code is widely available for a wide range of specific HMMs useful in bioinformatics problems

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Hidden Markov Models: CpG Islands

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Example (Durbin et al. (1998))

From the observed sequence alone, it is not possible to deduce whether a given CG pair is part of a CpG island, or is just a normal dinucleotide pair.

We can use an eight state hidden Markov model (HMM)

- A_+, C_+, G_+, T_+ are the states corresponding to nucleotides within a CpG region
- $A_{-}, C_{-}, G_{-}, T_{-}$ are the states corresponding to nucleotides outside of a CpG region

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Hidden Markov Models: CpG Islands

Example (Durbin et al. (1998))

CG or CpG dinucleotides are common in certain parts of the human genome.

Regions that are CpG rich are called CpG islands, and are typically up to 1000 bases long.

It is of biological interest to determine the locations of CpG islands, as these regions are often associated with, for example, promoter sites.

···· CGCGTACGCGCGTGAC ····

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Hidden Markov Models: CpG Islands

Example (Continued)

- we can augment these eight states by two further states for completeness
 - a begin state
 - an end state

These two states are "silent" as they do not contribute a nucleotide to the observed nucleotide sequence, but are useful for introducing gaps into the sequence.

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Example (Continued)

In the CpG island case, the emission probabilities are all either 1 or zero, for example

Parameters in the state transition matrix can be obtained

 $P(x_i = A | h_i = A_+) = 1$ $\Pr(x_i = C | h_i = A_+) = 0$

Hidden Markov Models: CpG Islands

and so on.

from training data.

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Hidden Markov Models: Parameter Estimation

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Hidden Markov Models: CpG Islands

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Example (Continued)

For the CpG example, within the islands

+	A	С	G	Т
Α	0.180	0.274	0.426	0.120
С	0.171	0.368	0.274	0.188
G	0.161	0.339	0.375	0.125
Т	0.079	0.355	0.384	0.182

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For HMMs, we need to estimate

transition probabilities

$$a_{kl} = \Pr(h_i = l | h_{i-1} = k)$$

emission probabilities

```
e_k(b) = \Pr(x_i = b | h_i = k)
```

where x_i is the nucleotide in position *i*.

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Statistical Hidden Markov Models: CpG Islands

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Example (Continued)						
Outside the i	slan	ds				
	-	A	С	G	т	
	Α	0.300	0.205	0.285	0.210	
	С	0.322	0.298	0.078	0.302	
	G	0.248	0.246	0.298	0.208	
	Т	0.177	0.239	0.292	0.292	

Need also to assess probabilities of transitions between region types, for example $A_+ \rightarrow C_-$.

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Example (Continued)

Hidden states reflect consensus sequence, and observed states differ due to evolutionary forces.

Can also build HMM to infer protein secondary structures

- α-helices
- β-sheets
- insertions/deletions
- loops
- · features and sites

Markov Models Monte Carlo

Т	
0.210	
0.302	
0.208	
0.292	
itions botwoon	

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Hidden Markov Models

More complicated examples can be handled using the same framework

Example (Globin Proteins)

HBA_HUMAN	GSAQVKGHGKKVADALTNAVAHVDDMPNALSALSDLHAHKL
. A. 198 J. 1	G+ +VK+HGKKV A++++AH+D++ ++++LS+LH KL
HBB_HUMAN	GNPKVKAHGKKVLGAFSDGLAHLDNLKGTFATLSELHCDKL
(b)	
HBA_HUMAN	GSAQVKGHGKKVADALTNAVAHVDDMPNALSALSDLHAH
	++ ++++H+ KV + +A ++ +L+ L+++H+ 1
LGB2_LUPLU	NNPELQAHAGKVFKLVYEAAIQLQVTGVVVTDATLKNLGSVHVS
(c)	(a) A set of the s
HBA HUMAN	GSAOVKGHGKKVADALTNAVAHVDDMPNALSALSDLHAHK
	GS+ + G + + D L + + H + D + A + AL D + + AH +
F11G11.2	GSGYLVGDSLTFVDLLVAOHTADLLAANAALLDEFPOFKAHO

Sequence Alignments to human alpha globin HBA_HUMAN; (a) human beta globin HBB_HUMAN, (b) lupin leghaemoglobin LGB2_LUPLU, (c) nematode glutathione S-transferase F11G11.2.

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Monte Carlo

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Monte Carlo is a simulation approach to studying biological models and systems that allows generation of hypothetical data.

- · Allows assessment of proposed model through simulation of possibly complicated systems
- Relies on random number generation techniques
- · Requires a stochastic (probability) model for the experimental variability or data generation model.

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Example

In statistical population genetics, a common model for the correlation in genomic sequences is the coalescent, which is based on a stochastic process model of evolution, mutation, selection and recombination.

It is a complicated model that is formulated via a tree model for the unobserved relationships. Analytic computations are difficult to carry out, but the model is relatively easy to simulate forward in time.

Therefore properties of the model can be studied using

- forward Monte Carlo simulation of replicate data
- summarization of the simulated data.

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• Statistical calculations are an essential part of bioinformatics and computational biology.

- Computational statistical methods such as the bootstrap forms a central part of statistical bioinformatics as they avoid the need for use of toy models.
- Current challenges largely involve the magnitude and diversity of biological data available, and the need for coherent data fusion.