MATH 556 - ASSIGNMENT 3

To be handed in not later than 11.59pm, 21st November 2022.
Please submit your solutions as pdf via myCourses.

1. Suppose that Z_1 and Z_2 are independent random variables having a $Normal(0,1)$ distribution.

   (a) Find the joint pdf of random variables X_1 and X_2 defined by

   $$X_1 = \frac{Z_1}{Z_2} \qquad X_2 = Z_1 + Z_2.$$

   5 Marks

   (b) Find the covariance between random variables Y_1 and Y_2 where

   $$Y_1 = Z_1 + Z_2 \qquad Y_2 = Z_1 - Z_2.$$

   2 Marks

   Are Y_1 and Y_2 independent ? Justify your answer. 1 Mark

   (c) Find the characteristic function of

   $$V = a_1 Z_1 + a_2 Z_2$$

   for real constants a_1 and a_2. 2 Marks

2. Suppose that $X = (X_1, X_2)^\top \sim Dirichlet(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = \alpha_2 = \alpha_3 = 2$.

   (a) Prove (showing your working) that marginally $X_1 \sim Beta(a, b)$, for a, b to be identified.

   3 Marks

   (b) Find the correlation between X_1 and V defined by

   $$V = 1 - X_1.$$

   3 Marks

3. Suppose that X and Y have joint distribution specified by

   $$X \sim Beta(1,1)$$
   $$Y|X = x \sim Binomial(n, x)$$

   for fixed $n \geq 1$. Find $\text{Var}_Y[Y]$. 4 Marks