MATH 556 - ASSIGNMENT 2

To be handed in not later than 11.59pm, 31st October 2022.
Please submit your solutions as pdf via myCourses.

1. Consider the discrete pmf, f_X, defined for $i = 1, 2, \ldots$ by

$$f_X(x_i) = p_i$$

where $\mathbb{X} = \{x_1, x_2, \ldots\}$, with $x_i > 0$ for all i. Suppose that $\mu = \mathbb{E}_X[X] < \infty$.

Show that

$$\mu e^{\mu} \leq \sum_{i=1}^{\infty} p_i x_i e^{x_i}.$$

4 Marks

2. For the following cfs, φ_X, find the corresponding distribution (by name, or in terms of the pmf, pdf or cdf), or demonstrate why the function is not a valid cf. You may quote results from the distributions formula sheet or from lectures.

(a) For $t \in \mathbb{R}$

$$\varphi_X(t) = \frac{2}{2 + t^2}$$

2 Marks

(b) For $t \in \mathbb{R}$

$$\varphi_X(t) = \frac{1}{2}(1 + \cos(t) + i \sin(t))$$

2 Marks

(c) For $t \in \mathbb{R}$

$$\varphi_X(t) = \frac{1}{2} e^{it} \left(1 + \exp\{e^{it} - 1 - it\}\right)$$

4 Marks

3. A key result for cfs is that if X and Y are independent, and $Z = X + Y$, then

$$\varphi_Z(t) = \varphi_X(t)\varphi_Y(t).$$

Does this result ever hold if $Z = X + Y$ but X and Y are **not** independent? Justify your answer.

4 Marks

4. Suppose that $X \sim Exponential(1)$ with cf denoted $\varphi_X(t)$, and that $\phi(\cdot)$ is the standard normal pdf. Consider the function

$$\varphi(t) = \int_{-\infty}^{\infty} \varphi_X(ts)\phi(s)\, ds.$$

Is $\varphi(t)$ a valid cf ? Justify your answer. 4 Marks