MATH 556 - EXERCISES 1

Not for assessment.

1. The absolutely continuous cdf F_X has density

$$f_X(x) = c\exp\{-\lambda|x-\theta|\} \qquad x \in \mathbb{R}$$

for parameters $\lambda > 0$ and $\theta \in \mathbb{R}$.

Compute

(i) The constant c;
(ii) The cdf F_X;
(iii) The quantile function Q_X;
(iv) The expectation $\mathbb{E}_X[X]$;
(v) The variance $\text{Var}_X[X]$.

2. A vector of d random variables are termed *independent* if

$$P_X\left[\bigcap_{j=1}^{d}(X_j \in A_j)\right] = \prod_{j=1}^{d} P_{X_j}[X_j \in A_j]$$

for all sets $A_1, \ldots, A_d \subset \mathbb{R}$. This statement can be interpreted equivalently using cdfs or pdfs.

Consider the joint density defined on the unit cube $(0,1)^3$.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = c(1-\sin(2\pi x_1)\sin(2\pi x_2)\sin(2\pi x_3))$$

and zero otherwise, for some constant c.

(a) Are (X_1, X_2) independent ?
(b) Are (X_1, X_2, X_3) independent ?

Justify your answers.

3. Suppose that $X_1, \ldots, X_n$ are independent and identically distributed standard normal random variables (Normal$(0,1)$). Consider the transformed random variables

$$S = \sum_{i=1}^{n} X_i^2 \qquad\qquad T_i = X_i^2/S \quad i = 1, 2, \ldots, n.$$

Show, using the general transformation approach, that S and $T = (T_1, \ldots, T_n)$ are independent, that is, the joint pdf of S and T factorizes into the marginal for S and the marginal for T for all arguments $(s,t) \in \mathbb{R}^{n+1}$.

4. Show that if $X \sim Pareto(\theta, \alpha)$ (parameterized as on the distributions handout), then

$$X \stackrel{d}{=} g(Z)$$

where $Z \sim Exponential(1)$, and $g(.)$ is some transformation to be found.