MATH 556 - ASSIGNMENT 4

To be handed in not later than 11.59pm, 3rd December 2019.
Please submit your solutions as pdf via myCourses.

1. Suppose $X_1, \ldots, X_n$ are independent $Normal(\mu, \sigma^2)$ rvs. Denote by $\overline{X}$ and s^2 the sample mean and sample variance statistics.

   (a) Derive the distribution of the rv
   $$T_1 = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$
   You may use without proof results from handouts concerning $\overline{X}$ and s^2, but must present details of the derivation for T_1. 4 Marks

   (b) By first considering its form for fixed finite n, derive the *limiting distribution* of s^2, that is, the probability distribution of s^2 as $n \longrightarrow \infty$. 2 Marks

   Hint: consider the expectation and variance of s^2.

   (c) Derive the limiting distribution of T_1 as $n \longrightarrow \infty$. 2 Marks

2. Suppose that for positive integers n_1 and n_2, rvs $V_1 \sim \chi^2_{n_1}$ and $V_2 \sim \chi^2_{n_2}$ are independent.

   (a) Derive using multivariate transformation techniques the distribution of
   $$T_2 = \frac{V_1/n_1}{V_2/n_2}.$$
   Show full details of the calculation. 4 Marks

   (b) Identify the limiting distribution (as defined in Q1) of T_2 as $n_2 \longrightarrow \infty$. 2 Marks

3. Suppose that X is a continuous random variable with cdf
   $$F_X(x) = \mathbb{1}_{(0,\infty)}(x) \left(\frac{x^2}{1+x^2} \right)^n$$
   where n is a positive integer.

   (a) Derive, for fixed $x \in \mathbb{R}$, $P_X[X > x]$ 1 Mark

   (b) Describe, for fixed $x \in \mathbb{R}$, the behaviour of $P_X[X > x]$ as $n \longrightarrow \infty$. 2 Marks

   (c) Describe, for fixed $y \in \mathbb{R}$, the behaviour of $P_Y[Y > y]$ as $n \longrightarrow \infty$ if Y is the random variable defined by
   $$Y = \frac{X}{\sqrt{n}}.$$
   3 Marks