MATH 556 - ASSIGNMENT 2

To be handed in not later than 11.59pm, 27th October 2019.
Please submit your solutions as pdf via myCourses.

1. Suppose that X has a finite mixture distribution with cdf

$$F_X(x) = \sum_{k=1}^{K} \omega_k F_k(x) \qquad x \in \mathbb{R}$$

where K is a positive integer, $F_1, \ldots, F_K$ are distinct cdfs, and $\omega_1, \ldots, \omega_K$ satisfy

$$0 < \omega_k < 1 \quad \text{for all } k \qquad \sum_{k=1}^{K} \omega_k = 1.$$

Find the characteristic function (cf) for X in terms of the cfs corresponding to $F_1, \ldots, F_K$. 4 Marks

2. A sufficient condition for a distribution defined on $\mathbb{R}$ to be (absolutely) continuous is that its cf $\varphi(t)$ satisfies

$$\int_{-\infty}^{\infty} |\varphi(t)| \, dt < \infty.$$

where $|\varphi(t)|$ is the modulus of the complex-valued quantity $\varphi(t)$. By finding a suitable counterexample, show that this is not a necessary condition for (absolute) continuity. That is, find an (absolutely) continuous distribution with cf $\varphi(t)$ for which

$$\int_{-\infty}^{\infty} |\varphi(t)| \, dt = \infty.$$

4 Marks

3. Suppose that cf $\varphi_X(t)$ takes the form

$$\varphi_X(t) = \frac{1}{2} \left(\cos(t) + \cos(\pi t) \right).$$

   (a) Is the distribution of X (absolutely) continuous ? Justify your answer. 2 Marks

   (b) Comment on the finiteness or existence of $\mathbb{E}_X[X^r]$ for $r \geq 1$. 2 Marks

4. Compute

   (a) The first four cumulants of the $Poisson(\lambda)$ distribution. 2 Marks

   (b) The first three cumulants of the $Normal(\mu, \sigma^2)$ distribution. 2 Marks

5. Consider the function

$$\varphi(t) = \frac{1}{(1 + 2t^2 + t^4)}.$$

Assess whether this function is a valid cf, and if it is valid, describe in as much detail as possible the distribution to which it corresponds. 4 Marks