MATH 556 - ASSIGNMENT 1

To be handed in not later than 11.59pm, 29th September 2019.
Please submit your solutions as pdf via myCourses.

1. Find the quantile function, $Q_X(p)$ for $0 < p < 1$, for the following cases:

   (a) X is distributed as $Weibull(3,2)$ (in the parameterization of the Distributions Formula Sheet).

   2 Marks

   (b) X has the discrete distribution with

   $$f_X(x) = c\mathbb{1}_{\{1,2,\ldots,10\}}(x) \qquad x \in \mathbb{R}$$

   for some c to be determined. 2 Marks

   (c) X has the distribution with

   $$F_X(x) = \begin{cases} 0 & x < 0 \\ \dfrac{1}{2} & 0 \leq x < 1 \\ \dfrac{3}{4} & x = 1 \\ (1 - c\exp(-(x-1)) & x > 1 \end{cases}$$

   for some c to be determined. 4 Marks

2. Suppose that X has a standard Normal distribution, $X \sim Normal(0,1)$ with pdf

   $$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} \qquad x \in \mathbb{R}$$

   Compute and sketch (or plot) the pdfs of the random variables

   (a) $Y_1 = X^2$ 1 Mark

   (b) $Y_2 = |X|$ 1 Mark

   (c) $Y_3 = 2X - X^2$ 2 Marks

   (d) $Y_4 = F_X(X)$, where $F_X(.)$ is the cdf of X. 2 Marks

3. Suppose that X is a continuous random variable with support $\mathbb{X} = \mathbb{R}$, and with cdf $F_X(x)$. Suppose that Y is a transformed variable given by

   $$Y = \{F_X(X)\}^k$$

   for positive integer k. Find the expectation of Y. 3 Marks

4. A random rectangle is to be defined in the following way: one corner is anchored at the origin, the next corner is at $(X,0)$, the next corner is at (X,Y) and the final corner is at $(0,Y)$, where X and Y are continuous random variables, independently drawn from the $Exponential(2)$ distribution (using the parameterization from the Distribution Formula Sheet).

   Find the expectation of the area of the random rectangle. 3 Marks