MATH 556 - ASSIGNMENT 2

To be handed in not later than 10pm, 20th October 2014.
Please submit your solutions as pdf via myCourses

1. (a) Suppose that $\{\varphi_k(t)\}_{k=1}^n$ is a sequence of characteristic functions, and $\{c_k\}_{k=1}^n$ is a sequence of nonnegative real valued constants, with

$$\sum_{k=1}^n c_k = 1.$$

Show that

$$\sum_{k=1}^n c_k \varphi_k(t)$$

is also a characteristic function, and identify the distribution to which it corresponds.

4 Marks

(b) Does the result in (a) extend to the case where $n \longrightarrow \infty$? Justify your answer.

2 Marks

(c) If

$$\varphi_1(t) = \exp(-4t^2) \qquad \varphi_2(t) = (3 + \cos(t) + \cos(2t))/5$$

identify the distribution with cf

$$\frac{\varphi_1(t) + \varphi_2(t)}{2}.$$

4 Marks

2. Suppose X_1 and X_2 are independent random variables, and suppose also that X_1 and $X_1 - X_2$ are independent. Show that

$$\mathrm{P}_{X_1}[\, X_1 = c \,] = 1$$

for some constant c.

Hint: write $X_2 = X_1 + (X_2 - X_1)$, and recall that if $\varphi(t)$ is an arbitrary cf, then $\varphi(t)$ is continuous for all t.

4 Marks

3. Suppose that mgf $M_X(t)$ is defined, for a suitable neighbourhood of zero $(-\delta, \delta)$, as

$$M_X(t) = \frac{9e^{-t}}{(3+2t)^2}.$$

Find an expression for $\mathbb{E}_X[X^r]$, for $r = 1, 2, \ldots,$.

3 Marks

4. Suppose that $X \sim \text{Binomial}(n, \theta)$ for integer $n \geq 1$, and $0 < \theta < 1$. Let

$$Z_n = \frac{(X - n\theta)}{\sqrt{n\theta(1-\theta)}}.$$

Find the first two non-zero terms in the power series expansion of the cumulant generating function of Z_n, and the order of approximation (in terms of n) when truncating the expansion at the second term, for large n.

Recall that

$$\log\{(1+z)^n\} = n\{z - z^2/2 + \cdots\}.$$

3 Marks