Jensen’s Inequality - if \( g(x) \) is convex, so that for \( 0 < \lambda < 1 \),
\[
g(\lambda x + (1 - \lambda)y) \leq \lambda g(x) + (1 - \lambda)g(y)
\]
for all \( x \) and \( y \), then if \( X \) is a random variable with expectation \( \mu \),
\[
E_{f_X} [g(X)] \geq g(E_{f_X} [X])
\]
- extends to the multivariable case in a number of ways.

- If \( X \) is a \( k \)-dimensional vector random variable, but \( g \)
\[
g : \mathbb{R}^k \rightarrow \mathbb{R}
\]
is a convex scalar function, then
\[
E_{f_X} [g(X)] \geq g(E_{f_X} [X])
\]
for which the proof is similar to the original version.

- If \( g \)
\[
g : \mathbb{R}^k \rightarrow \mathbb{R}^d
\]
is a vector function, then the above result can be applied componentwise to the elements
\[
(g_1(x), g_2(x), \ldots, g_d(x))^T.
\]

- If \( g \) is a matrix-valued function, for example
\[
g(x) = xx^T
\]
then we can also consider matrix-type inequalities; for example, for two \( k \times k \) matrices, \( \Sigma_1 \) and \( \Sigma_2 \), we might write
\[
\Sigma_1 \geq \Sigma_2 \quad \text{if} \quad \Sigma_1 - \Sigma_2 \text{ is positive definite}
\]
that is, if
\[
x^T(\Sigma_1 - \Sigma_2)x \geq 0 \quad \forall x \in \mathbb{R}^k
\]
and legitimately write, say,
\[
E_{f_X} \left[ XX^T \right] \geq \mu\mu^T
\]
where \( \mu = E_{f_X} [X] \). However, general results relating to convex multivariable functions go beyond the scope of the course.