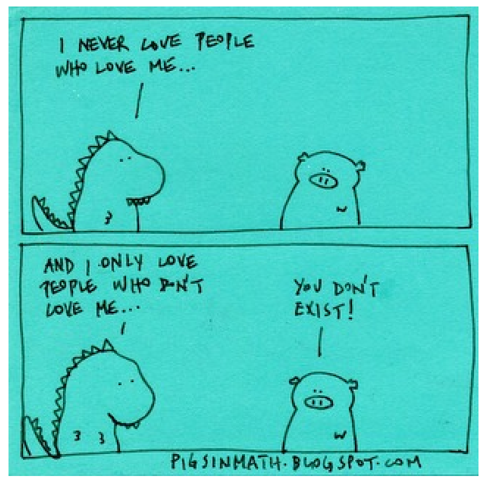
189-235A: Algebra 1

Practice Midterm Exam

1. Recall that the *power set* of a set A, denoted $P(A)$, is the set of all subsets of A. Let $f : A \to P(A)$ be a function from A to its power set. Show that the subset B of A defined by

$$B = \{a \in A \text{ such that } a \notin f(a)\}$$

is not an element of the image of f.



2. Compute the greatest common divisor of the integers

$$\begin{aligned} a &= 1320000000815000000000000000000132, \\ b &= 1320000000815. \end{aligned}$$

3. Compute the least residue modulo $N = 95$ of the integer

$$3^{1100000000000000000000000000000000000000000000000}$$

4. Using only the basic properties of the gcd proved in class, (and *not* the fundamental theorem of arithmetic) show that if a p is a prime and p divides a product ab of two integers, then p necessarily divides either a or b.