

189-726A: L -functions and Modular Forms

Assignment 3

Due: Wednesday, October 19

1. Let w_N be the Fricke involution that was introduced in class, acting on modular forms of weight k .

a) Show that, if f belongs to $S_k(\Gamma_0(N), \chi)$, then $w_N f$ belongs to $S_k(\Gamma_0(N), \bar{\chi})$.

b) Conclude that, if f is a *newform*, then there is a complex scalar w_f of norm one such that

$$w_N f = w_f \cdot \bar{f},$$

where \bar{f} denotes the modular form obtained from f by conjugating its Fourier expansion.

c) Use the result of part b) to write down a functional equation relating $L(f, s)$ to $L(\bar{f}, s)$.

2. Let \mathbf{T} be the \mathbf{Q} -algebra of Hecke operators T_n acting on $S_{24}(\mathbf{SL}_2(\mathbf{Z}))$. Express \mathbf{T} as a product of specific number fields, and write down a basis of eigenforms for $S_{24}(\mathbf{SL}_2(\mathbf{Z}))$. (This fun calculation was done by Hecke.)