

189-346/377B: Number Theory

Assignment 6

Due: Monday, March 26

1. Let p be an odd prime. Show that -2 is a quadratic residue modulo a prime p if and only if p is a prime of the form $m^2 + 2n^2$.
 2. Use question 1 and quadratic reciprocity to get a complete characterisation of all the integers that are of the form $m^2 + 2n^2$.
 3. Repeat questions 1 and 2 with $m^2 + 2n^2$ replaced by $m^2 + 3n^2$.
 4. Show that there are primes p for which -5 is a quadratic residue modulo p , yet which are not of the form $m^2 + 5n^2$.
 5. Make a list of the integers ≤ 100 that can be written in the form $m^2 + 5n^2$, and $2m^2 + 2mn + 3n^2$. Can you formulate some conjectures about how these sets of integers behave? (You may find it useful to write each integer in factored form.)
 6. Section 7.3, Problem 7 in Leveque.
 7. Section 8.4, Problem 4 in Leveque.
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8. Section 8.4, Problem 5 in Leveque.