PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 28, No. 1, April 1971

EXTENSIONS OF DERIVATIONS

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ABSTRACT. We show that for a class of algebras including separable algebras one can extend derivations of the center to derivations of the algebra.

The following theorem was proved in the special cases that C is a field by Hochschild [**Ho**] and C is a semilocal ring by Roy and Sridharan [**R, S**] [and for any C in [**Kn**]]. It is also a trivial consequence of a more general result proved by a short cohomological argument.

THEOREM 1. *Let A be an algebra separable over its center C and M be an $A \otimes_C A^{\mathrm{op}}$-module. Then any derivation $d: C \to M^A$ extends to a derivation $\tilde{d}: A \to M$.*

Since an algebra separable over its center C is C-projective [**A, G**, p. 379], Theorem 1 follows from

THEOREM 2. *Let A be a C-algebra, projective over C, of Hochschild dimension one and let M be an $A \otimes_C A^{\mathrm{op}}$-module. Then any derivation $d: C \to M^A$ extends to a derivation $\tilde{d}: A \to M$.*

PROOF. Let B be the split extension of A by M. That is, B is the additive group $A \oplus M$ with $(a, m)(a', m') = (aa', am' + ma')$. If we let C operate on B by $c(a, m) = (ca, cm + dc \cdot a) = (a, m)c$, then B is a C-algebra and the projection of B to A is a C-algebra homomorphism. It is also C-linearly split since A is C-projective. Thus the extension is an element of $H_C^2(A, M)$ which is zero by hypothesis. This means that there is a C-algebra splitting of $B \to A$, the second coordinate of which is easily seen to be a derivation extending d.

COROLLARY 1. *Let A, C, M be as above and A_0 be a separable C-subalgebra of A. Then any derivation $d: A_0 \to M$ which takes C to M^A can be extended to a derivation $\tilde{d}: A \to M$.*

PROOF. First restrict to C, then extend to A. The difference, on A_0, is C-linear and hence inner.

Received by the editors June 22, 1970.

AMS 1970 *subject classifications*. Primary 16A16, 16A62, 16A72, 18H15.

Key words and phrases. Separable algebras, Azumaya algebras, algebras of Hochschild dimension one, derivations, extension of derivations.

COROLLARY 2. *Let A be a R-algebra separable over its center C and let M be a $A \otimes_C A^{\mathrm{op}}$-module. Then* $H_R^1(A, M) \cong \mathrm{Der}_R(C, M^A)$.

PROOF. It is evident that any derivation of A to M restricts to a derivation $C \to M^A$.

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