Tutorial 5

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2020-02-10

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive

Two events A and B are mutually exclusive if $A \cap B = \emptyset$.

Independence

Two events A and B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

Note that it is possible for two events to be mutually exclusive but still dependent. For example, drawing cards one by one from a deck of 52 without replacement and denote the event that the first card is a king of spade by A, and the second card is a king of spade by B. Clearly $A \cap B = \emptyset$, but A and B are dependent.

Example 1

Biologists applied two methods (A and B) to data collected on proteins. The probability that protein is cross-referenced by method A is 0.41, by method B is 0.42, and by both methods is 0.4.

(a)

Find the probability that protein is cross-referenced by either method.

Solution:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.41 + 0.42 - 0.4 = 0.43$$

(b)

Find the probability that protein is not cross-referenced by either method.

Solution:

$$P((A \cup B)^C) = 1 - P(A \cup B) = 1 - 0.43 = 0.57$$

(c)

Are the two events A and B mutually exclusive? Are they independent?

Solution:

No, they are not mutually exclusive, because $A \cap B \neq \emptyset$.

$$P(A) \cdot P(B) = 0.42 \times 0.41 \neq 0.4 = P(A \cap B)$$

Therefore, they are not independent.

Example 2

An experiment is conducted with the following probabilities of outcomes, with two factors light (on or off), temparature (low, medium, or high):

```
##      Low Medium High
## On  0.50   0.10 0.05
## Off 0.25   0.07 0.03
```

Let A denote the event that light is on, B the event that light is on or temparature is medium, C the event that light is off and temparature is low, and D the event that temparature is high. Find the probabilities of the following events: A, B, C, D, A^C, $A \cup B$, $A \cap B$.

Solution:

$$P(A) = 0.5 + 0.1 + 0.05 = 0.65$$
$$P(B) = 0.65 + 0.10 + 0.07 - 0.10 = 0.72$$
$$P(C) = 0.25$$
$$P(D) = 0.05 + 0.03 = 0.08$$
$$P(A^C) = 1 - P(A) = 1 - 0.65 = 0.35$$

Since A is a subset of B, $A \cup B = B$, and $A \cap B = A$, we have the following:

$$P(A \cup B) = P(B) = 0.72$$

$$P(A \cap B) = P(A) = 0.65$$

Exampe 3

Data of 167 initiated deer buck encounters is recorded below. Answer the following questions.

```
##          Initiator Wins No Winner Initiator Loses Total
## Fight                26        23              15    64
## No Fight             80        12              11   103
## Total               106        35              26   167
```

(a)

What is the probability that fight happens and initiator wins?

Solution:

Let F denote the event that there was a fight, and $\bar{F}$ the event that there was no fight. Also let W denote the event that initiator wins, N the event that there is no winner, and L the event that initiator loses.

$$P(F \cap W) = \frac{26}{167}$$

(b)

What is the probability that no fight occurs?

Solution:

$$P(F^C) = \frac{103}{167}$$

(c)

What is the probability that there is no winner?

Solution:

$$P(N) = \frac{35}{167}$$

(d)

What is the probability that fight occurs or initiator loses?

Solution:

$$P(F \cup L) = P(F) + P(L) - P(F \cap L) = \frac{64}{167} + \frac{26}{167} - \frac{15}{167} = \frac{75}{167}$$