Tutorial 4

Xiaonan Da

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Example 1: Coin Flip

Flip a fair coin 3 times and record the outcomes. Denote Heads as H, and Tails as T. We can assign numerical values to H and T, with 1 and 0 respectively. Identify the following and compute their probabilities: the sample space, the event A_i that the i^{th} flip is Heads, the event that at least one flip is Heads, the event that all flips are Heads, the event that there were at least two consecutive Heads, and the event that there were exactly two consecutive Heads.

Solution:

Intuively,

$$P(A) = \frac{|A|}{|S|}$$

The sample space is

$$S = \{(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)\}$$

An event A is a subset of the sample space S.

The event that the first flip is Heads is

$$A_1 = \{(HHH), (HHT), (HTH), (HTT)\}$$

$$P(A_1) = \frac{|A_1|}{|S|} = \frac{4}{8} = \frac{1}{2}$$

The event that the second flip is Heads is

$$A_2 = \{(HHH), (HHT), (THH), (THT)\}$$

$$P(A_2) = \frac{|A_2|}{|S|} = \frac{4}{8} = \frac{1}{2}$$

The event that the third flip is Heads is

$$A_3 = \{(HHH), (HTH), (THH), (TTH)\}$$

$$P(A_3) = \frac{|A_3|}{|S|} = \frac{4}{8} = \frac{1}{2}$$

The event that at least one flip is Heads is

$$B = A_1 \cup A_2 \cup A_3 = \cup_{i=1}^{3} A_i = \{(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH)\}$$

$$P(B) = \frac{|B|}{|S|} = \frac{7}{8}$$

The event that all flips are Heads is

$$C = A_1 \cap A_2 \cap A_3 = \cap_{i=1}^{3} A_i = \{(HHH)\}$$

$$P(C) = \frac{|C|}{|S|} = \frac{1}{8}$$

The event that there were at least two consecutive Heads is

$$D = \cup_{i=1}^{2}(A_i \cap A_{i+1}) = (A_1 \cap A_2) \cup (A_2 \cap A_3) = \{(HHH), (HHT), (THH)\}$$

$$P(D) = \frac{|D|}{|S|} = \frac{3}{8}$$

The event that there were exactly two consecutive Heads is

$$E = D \setminus C = D \cap C^C = \{(HHT), (THH)\}$$

$$P(E) = \frac{|E|}{|S|} = \frac{1}{4}$$

Example 2: Coin Flip Continued

Flip a fair coin 10 times, intuitively how many Heads and Tails do you expect to see? Verify your answer by random sampling in R.

Solution:

```
# Coin Toss
k <- 10
# Toss a fair coin 10 times.
# Perform the experiment
# make k=10 choice from n=2 objects, {1,2}, one at a time,
# with replacement.

# The following code generate a sequence of length 10,
# representing the result of 10 tosses.
coin.flip <- sample(2,k,replace=TRUE)
# The outcome of the experiment
coin.flip
```

```
##  [1] 1 2 1 1 2 2 2 1 1 1
```

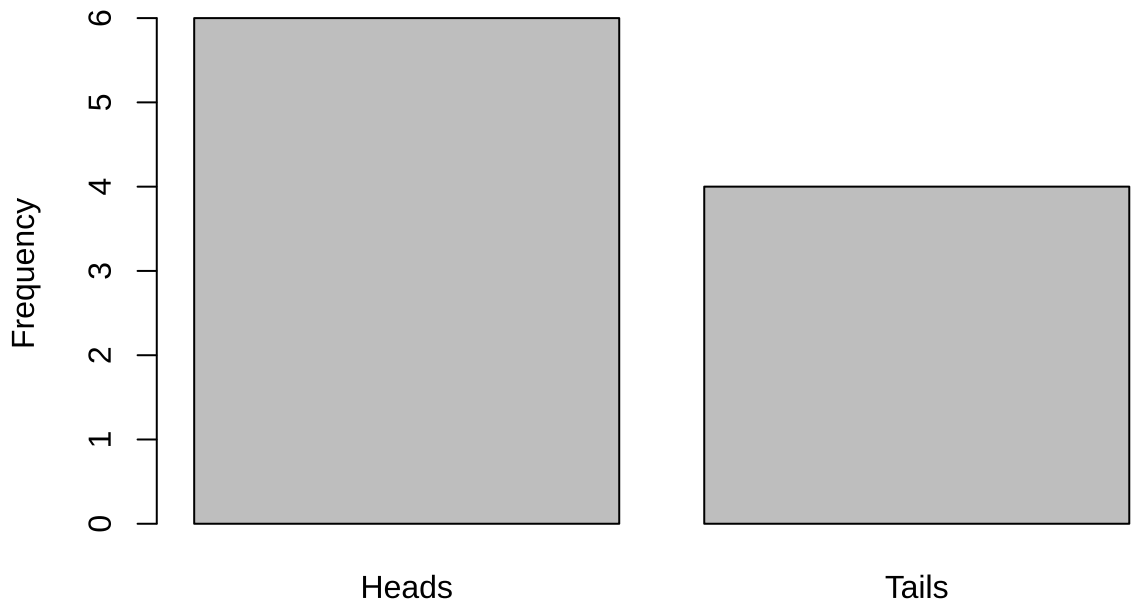
```
# number of Heads
nhead  <- sum(coin.flip==1)
nhead
```

```
## [1] 6
```

```
# number of Tails
ntail  <- sum(coin.flip==2)
ntail
```

```
## [1] 4
```

```
# make a barplot for the results
barplot(table(coin.flip), names.arg = c("Heads","Tails"), ylab = "Frequency")
```



Example 3: Dice Rolling

Roll two fair dice once and record the outcomes. Identify the following and compute their probabilities: the sample space, the event that the sum of the two numbers is 6, the event that both numbers are even, and the event that both dice come up number 2.

Solution:

The sample space is

$$S = \{(1,1),(1,2),\cdots,(1,6),(2,1),(2,2),\cdots,(2,6),\cdots(6,1),(6,2),\cdots,(6,6)\}$$

The event that the sum of the two numbers is 6

$$A = \{(1,5),(5,1),(2,4),(4,2),(3,3)\}$$

$$P(A) = \frac{|A|}{|S|} = \frac{5}{36}$$

The event that both numbers are even is

$$B = \{(2,2),(2,4),(4,2),(2,6),(6,2),(4,4),(4,6),(6,4),(6,6)\}$$

$$P(B) = \frac{|B|}{|S|} = \frac{1}{4}$$

The event that both dice come up 2 is

$$C = \{(2,2)\}$$

$$P(C) = \frac{1}{36}$$

Example 4: Dice Rolling Continued

Roll two fair dice 100 times, intuitively how many times do you expect to observe the same numbers on both dice? Verify your answer by random smapling in R.

Solution:

Let A_i denote the event that both numbers are i, $i = 1, 2, \cdots, 6$.

$$P(A_i) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(\cup_{i=1}^{6} A_i) = \sum_{i=1}^{6} P(A_i) = \frac{1}{6}$$

```
n <- 6
k <- 100
# Dice Rolling
# roll two fair dice 100 times
die1 <- sample(n,k,replace=TRUE)
die2 <- sample(n,k,replace=TRUE)
# if you want to ombine two columns of outcomes
dice <- cbind(die1,die2)
# the result of the experiment
dice
```

```
##        die1 die2
##   [1,]    1    3
##   [2,]    1    2
##   [3,]    6    4
##   [4,]    3    6
##   [5,]    4    3
##   [6,]    3    2
##   [7,]    4    6
##   [8,]    6    2
##   [9,]    3    6
##  [10,]    4    5
##  [11,]    2    2
##  [12,]    2    4
##  [13,]    2    2
##  [14,]    5    1
##  [15,]    2    2
##  [16,]    5    4
##  [17,]    3    5
##  [18,]    5    3
##  [19,]    1    3
##  [20,]    6    4
##  [21,]    5    2
##  [22,]    3    5
##  [23,]    6    6
##  [24,]    4    4
##  [25,]    1    6
##  [26,]    1    6
##  [27,]    1    1
##  [28,]    2    4
##  [29,]    4    3
##  [30,]    6    1
##  [31,]    4    3
##  [32,]    2    4
##  [33,]    4    5
```

```
## [34,]    4    5
## [35,]    1    3
## [36,]    4    4
## [37,]    6    5
## [38,]    5    2
## [39,]    1    2
## [40,]    3    4
## [41,]    1    5
## [42,]    5    2
## [43,]    2    3
## [44,]    6    2
## [45,]    5    3
## [46,]    1    3
## [47,]    3    6
## [48,]    6    2
## [49,]    6    5
## [50,]    5    3
## [51,]    5    1
## [52,]    2    5
## [53,]    3    1
## [54,]    4    4
## [55,]    2    2
## [56,]    2    1
## [57,]    5    4
## [58,]    5    1
## [59,]    4    3
## [60,]    3    1
## [61,]    5    3
## [62,]    2    3
## [63,]    6    4
## [64,]    6    3
## [65,]    3    2
## [66,]    4    4
## [67,]    2    4
## [68,]    2    3
## [69,]    2    2
## [70,]    3    1
## [71,]    6    4
## [72,]    1    1
## [73,]    1    6
## [74,]    4    1
## [75,]    3    1
## [76,]    1    4
## [77,]    6    2
## [78,]    4    2
## [79,]    4    6
## [80,]    5    5
## [81,]    3    3
## [82,]    6    6
## [83,]    4    6
## [84,]    2    3
## [85,]    6    2
## [86,]    6    2
## [87,]    3    5
```

```
## [88,]    5    3
## [89,]    5    1
## [90,]    3    6
## [91,]    1    3
## [92,]    1    1
## [93,]    1    4
## [94,]    3    6
## [95,]    5    6
## [96,]    6    4
## [97,]    4    3
## [98,]    1    6
## [99,]    3    1
## [100,]   2    6
```

```r
# number of times that the same numbers come up on both dice
sum(die1==die2)
```

```
## [1] 16
```