Math 340: Discrete Structures II *Thursday, February 13th, 2014*

Midterm Exam

Instructions. The exam is 80 minutes long and contains 3 questions. Write your answers clearly in the notebook provided. [You may quote any result/theorem seen in the lectures or in the assignments without proving it (unless, of course, that is what you are asked to prove).]

1. *Matchings.* Take a bipartite graph $G = (V, E)$ where the two parts of V in the bipartition are X and Y, where $|X| = |Y| = n$.

   (a) State Hall's Theorem. [2 marks]

   (b) Let the bipartite graph G be *connected* and have maximum degree 2. Explain why (without using Hall's Theorem) G must contain a perfect matching. [4 marks]

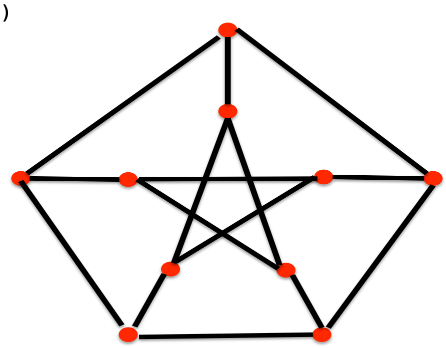
   (c) Now prove the result in (b) using Hall's Theorem. [4 marks]

2. *Planar Graphs.*

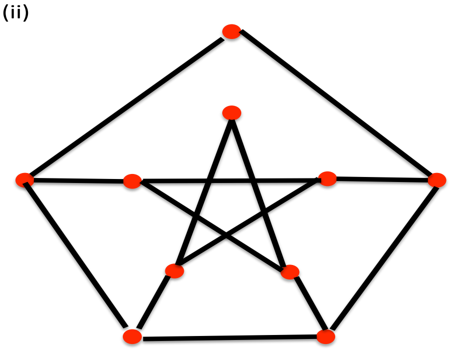
   (a) State Kuratowski's theorem. [2 marks]

   (b) Explain whether or not each of the following two graphs is planar. [8 marks]

(i)



(ii)



3. *Planar Graphs.*

(a) State Euler's Formula for planar graphs. [2 marks]

(b) Use Euler's Formula to give an upper bound on the number of edges (in terms of the number of vertices) in G. [4 marks]

(c) Let $\bar{G} = (V, \bar{E})$ be the *complement* of $G = (V, E)$.[1] Prove that at least one of G or $\bar{G}$ is not planar if $|V| = n \geq 11$. [4 marks]

[1] Recall that $\bar{G} = (V, \bar{E})$ is the *complement* of $G = (V, E)$ if (i, j) is an edge in $\bar{G}$ if and only if (i, j) is not an edge in G.