

Mathematics 366 Sample Problems

Carefully justify your answer in each of the following problems.

1) Compute

$$\int_0^{2\pi} \frac{1 - \frac{1}{4} \cos \theta}{1 + \frac{1}{16} \cos^2 \theta} d\theta.$$

Please justify your answer.

2) Let C be a simple closed contour and let z_1 and z_2 lie inside C . Compute

$$\frac{1}{2\pi i} \oint_C \frac{\sin z}{(z - z_1)^2 (z - z_2)} dz.$$

Please justify your answer.

3) Let $f(z) = e^x + ie^{2y}$ where $z = x + iy$ is a complex variable defined in the whole complex plane. For what values of z does $f'(z)$ exist?

4) Show that $u(x, y) = e^x \cos y + e^y \cos x + xy$ is harmonic. Compute a holomorphic function $f(z)$ in the plane with the property that $u = \Re f$.

5) (a) Find $\int_{1+i}^{-1-i} \frac{\log z}{z} dz$ where the integral is along a contour not intersecting the branch cut for the principal logarithm $\log z$.

(a) Compute $\int_{|z|=1} \frac{\log z}{z} dz$.

6) (a) Show that all the roots of $z^4 + z^3 + 1 = 0$ lie inside $|z| = \frac{3}{2}$.

(b) Show that all the roots in (a) lie outside $|z| = \frac{3}{4}$.

7) Compute $\max_{|z| \leq 1} |f(z)|$ where $f(z) = \frac{\sin z}{z}$.

8) Evaluate the following integral using residues:

$$\int_{-\infty}^{\infty} \frac{x[\sin^2(2x) + \cos(2x)]}{x^2 + 3} dx.$$