Mathematics 355 : Homework # 4

Due (in class): Monday, April 15.

1) Suppose $f \in C^0(\mathbf{R})$ and periodic with $f(x+2\pi) = f(x)$. Suppose, in addition, that f is continuously differentiable at $x_0 \in [-\pi, \pi]$. Prove that

$$f(x_0) = \lim_{N \to \infty} \sum_{n=-N}^{N} a_n e^{inx_0},$$

where $a_n = (2\pi)^{-1} \int_{-\pi}^{\pi} f(t) e^{-int} dt$.

2) Consider the *Volterra* integral operator $T : L^2([0,1]) \to L^2([0,1])$ of the form

$$Tf(x) = \int_0^x f(y) dy, \;\; x \in [0,1].$$

Prove that T is a compact operator and that the spectrum $\sigma(T) = \{0\}$.

The following problems are from the text:

1. pgs. 253-258: # 2, 6, 7 (a)-(d), 13, 15, 16, 18 (a)-(c).