Mathematics 355 : Homework # 2

Due (in class): Monday, February 18.

1. For $\omega \in I = [0,1]$ let $R_k(\omega); k = 1, 2, 3, ..$ denote the Rademacher functions and $S_n(\omega) = \sum_{k=1}^{n} R_k(\omega)$. For each $n \geq 1$ consider the measurable set

$$B_n = \{\omega \in I; |S_n(\omega)| > \epsilon n\}.$$

We showed in class that

$$m(B_n) \leq \frac{3}{\epsilon^4 n^2}.$$

By choosing $\epsilon = \epsilon(n)$ depending on n appropriately, show that for

$$N = \{\omega \in I; \lim_{n\to\infty} \frac{S_n(\omega)}{n} = 0\}$$

one has

$$m(N^c) = 0.$$

This completes the proof of the strong law of large numbers.

The following problems are from the text:

1. pgs. 90-93: # 4, 5 (a)-(c), 6 (a)-(b), 8, 10, 15, 17 (a)-(c), 19.